Error Propagation in Macro Transport Models

La propagation d'erreurs dans les macro modèles de transport

Fehlerfortpflanzung in Makroverkehrsmodellen

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Forschungsauftrag SVI 2004/015 auf Antrag der Vereinigung Schweizerischer Verkehrsingenieure (SVI)

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CONTENT

Summary

The Problem can be defined with a few simple questions:

- How do input errors propagate through transport models?
- How do these input errors influence the output errors?
- Which algorithms minimize these influences?

All error analysis methods that we encountered during the literature search use some form of repeated simulation with random variation of input variables. This method is limited because input errors cannot be traced explicitly through all calculations.

The approach of this study is radically different. A newly created transport modeling software package includes explicitly coded error propagation rules using “operator overloading”. This functionality, which is available in some high level languages like ADA or C++, allows to overload operators like +, -, *, /, exp, etc. with operations defined by the programmer. The program recognizes automatically which operation it has to execute from the type of input data.

This newly created software tool was used to execute all calculations and tests for this study. Each test consists of one or more of the four steps of the four step model and a set of input variables with errors varying within realistic ranges.

A first series of tests was set up with a small synthetic model. The aim was to analyze the individual steps of the classic four step model, combination of them, as well as two matrix calibration methods. A second series of tests was applied to the Swiss transport model to check if the conclusion drawn from the small synthetic model were equally valid for the assignment and the two matrix calibration methods when applied to large real cases.

The results for the four step model show that generation, distribution and assignment do not amplify input errors, whereas modal split can multiply the input error by a factor of 3 to 4.

The two input variables having the most influence on output error are trip values and link travel times. The errors of trip values influence the errors of link values and the errors of link travel times influence the errors of trip travel time.

The major result of the study concerns the two calibration methods. One of them is the classical Entropy method of Willumsen [Ortuzar, Willumsen]. The other one is its additive form developed nearly 20 years ago by [de Rham 87]. Both methods produce best possible estimates for link loads, but the second one with much less distortion of the resulting distance distribution.

The comparison of both calibration methods for the Swiss model led to the discovery that the additive method produces link flow errors about 6 times smaller and trip travel time errors about 3 times smaller than with the classic one.

This finding provides an end to an old controversy between users of the Entropy method, who intuitively felt that it produced results with great variability, and users of the additive form who had no means to prove that their method was the more reliable one.

The two main recommendations to diminish modeling errors are:

- give priority to diminish the input errors of trip values and link travel times;
- use the additive Entropy method instead of the classic multiplicative one.

The combination of the two recommendations ensures that the investment in better trip values and link travel times will not be destroyed by bad behavior in error propagation of the classical multiplicative Entropy method.

Error propagation produces new solutions to the problem of quality management of online transport modeling. The old method of repeated simulation implies additional software to manage inputs and outputs, multiple runs to ensure significance as well as knowledge of multivariate statistics to analyze the results. The consequence is that quality control is rarely applied.

All these disadvantages fade away by explicitly integrating error propagation rules into the modeling software. The input errors from measured flows and speeds propagate through all calculation and produce output errors of calculated flows and speeds on all links of the network.

This offers new possibilities for the quality monitoring of online applications. Weak points of the measurement infrastructure are rapidly targeted and eliminated. Algorithms are chosen to minimize the numerical effects of error propagation. The quality of the estimated traffic situation, short term forecasts, traffic information and travel times is controlled and guaranteed 24 hours a day and 7 days a week.

Résumé

Le problème peut être défini à l'aide de quelques questions simples:

- comment est-ce que les erreurs se propagent à travers un modèle de transport?
- quelle est l'influence des erreurs d'entrée sur les erreurs de sorties?
- quels sont les algorithmes qui minimisent ces influences?

Toutes les méthodes d'analyse d'erreurs rencontrées lors de la recherche bibliographique utilisent une forme ou une autre de simulation répétée en faisant varier aléatoirement des données d'entrée. Cette méthode est limitée parce qu'elle ne permet pas de suivre explicitement les erreurs d'entrée tout au long des calculs.

L'approche de cette étude est radicalement différente. Un nouveau logiciel de modélisation des transports a été créé, dans lequel les lois de propagation d'erreurs sont codées explicitement à l'aide de la surcharge d'opérateurs. Cette fonctionnalité, disponible dans certains langages de haut niveau tel que ADA ou C++, permet de surcharger des opérateurs tels que +, -, *, /, exp, etc. avec des opérations définies par le programmeur. Le logiciel reconnaît automatiquement quelle opération il doit effectuer à partir du type des données d'entrée.

C'est cet outil nouvellement créé qui a été utilisé pour tous les calculs et tests de cette étude. Chaque test consiste en une méthode soumise à l'analyse avec un ensemble de variables d'entrées dont les erreurs peuvent varier dans des limites réalistes.

Une première série de tests a été appliquée à un petit modèle synthétique. Le but était d'analyser les étapes du modèle classique à quatre étapes, certaines de leurs combinaisons, ainsi que deux méthodes de calage de matrices. Une seconde série de test appliquée au modèle suisse de transport a permis de contrôler si les résultats obtenus étaient également valables pour l'affectation et les deux méthodes de calage appliquées à de grands exemples réels.

Les résultats pour le modèle à quatre étapes montrent que la génération, la distribution et l'affectation n'augmentent pas les erreurs d'entrée, alors que la répartition modale peut multiplier l'erreur par un facteur de 3 à 4.

Les deux variables d'entrée influençant le plus les erreurs de sortie sont les valeurs des déplacements et les temps de parcours des tronçons. Les erreurs des valeurs de déplacements influencent les erreurs des charges des tronçons et les erreurs des temps des tronçons influencent les erreurs des temps de parcours des déplacements.

Un résultat majeur de cette étude concerne les deux méthodes de calage. L'une est la méthode classique de l'Entropie selon Willumsen [Ortuzar, Willumsen]. L'autre a été développée il y a près de 20 ans par [de Rham 87]. Les deux méthodes produisent les meilleurs estimations possibles des charges des tronçons, mais la deuxième avec une moindre distorsion de la distribution des distances.

La comparaison des deux méthodes appliquées au modèle suisse a démontré que la méthode additive produit des erreurs de charges des tronçons environs 6 fois plus petites et des erreurs de temps de parcours environs 3 fois plus petites qu'avec la méthode classique.

Ce résultat marque la fin d'une vielle controverse entre utilisateurs de l'Entropie se doutant intuitivement de la variabilité de leurs résultats et ceux de la méthode additive ne pouvant pas démontrer la meilleure fiabilité des leurs.

Les deux recommandations principales pour diminuer les erreurs des modèles sont:

- diminuer en priorité les erreurs des déplacements et des temps des tronçons ;
- appliquer la variante additive de l'Entropie au lieu de la méthode multiplicative.

La combinaison des deux recommandations évite même que l'investissement dans de meilleurs déplacements et temps des tronçons soit annulé par le mauvais comportement de la propagation d'erreurs dans méthode classique de l'Entropie.

La propagation d'erreur permet également de trouver de nouvelles solutions au problème de la gestion de la qualité de résultats pour la modélisation en ligne. L'ancienne méthode des simulations répétées impliquait un surplus de programmation pour gérer les entrées et les sorties, de multiples essais pour assurer la représentativité ainsi que des connaissances en statistique multivariée pour l'analyse des résultats. La conséquence est simple : le suivi de qualité n'est pas appliqué.

Ces désavantages disparaissent lorsque l'on intègre les règles de propagation d'erreurs dans la modélisation de façon explicite. Les erreurs de débits et de vitesses mesurés à l'entrée se propagent à travers le modèle et produisent les erreurs de débits et de vitesses calculés en sortie pour chaque arc du réseau.

Cela ouvre de nouvelles perspectives pour une gestion (monitoring) efficace de la qualité d'applications en ligne. Les points faibles de l'infrastructure de mesure sont rapidement identifiés et éliminés. Les algorithmes de calage sont choisis afin de minimiser les effets numériques de la propagation d'erreurs. La qualité de la situation de trafic, des prévisions à court terme, de l'information routière ainsi que des temps de parcours est surveillée et garantie 24 heure sur 24 et 7 jours sur 7.

Zusammenfassung

Das Problem lässt sich mit einfachen Fragen definieren:

- Wie pflanzen sich Input-Fehler in Verkehrsmodellen fort?
- Welchen Einfluss habe diese Input-Fehler auf die Output-Fehler?
- Welche Algorithmen minimieren diesen Einfluss?

Alle in der Literatursuche gefundenen Methoden nutzen eine Art der wiederholten Simulation, wobei der Wert von Eingangsgrössen zufällig geändert wird. Diese Methode ist begrenzt, weil Eingangsfehler während den Berechnungen nicht explizit verfolgt werden können.

In dieser Studie ist der gewählte Ansatz radikal anders. Eine neue Verkehrsmodellsoftware wurde entwickelt, in der die Gesetze der Fehlerfortpflanzung mit der Technik der „Operator-Überladung" explizit programmiert sind. Diese Funktionalität, die in gewissen Hochsprachen wie ADA oder C++ zur Verfügung steht, erlaubt, Operatoren wie +, -, *, /, exp, usw. mit eigenem Code zu überladen. Das Programm erkennt automatisch die durchzuführende Operation aus dem Typ der Eingangsgrössen.

Mit diesem neuen Werkzeug wurden alle Berechnungen und Tests dieser Studie durchgeführt. Jeder Test besteht aus einem oder mehreren Schritten des Vier-Schritt-Modells und einer Menge Eingangsgrössen, deren Fehler sich in realistischen Grenzen bewegen können.

Eine erste Serie Tests wurde mit einem kleinen synthetischen Modell durchgeführt. Das Ziel war, die Schritte des klassischen Vier-Schritt-Modells, Kombinationen davon, sowie zwei Kalibrierungsmethoden zu analysieren. Eine zweite Serie Tests mit dem Schweizer Verkehrsmodell (ARE-Modell) diente zur Kontrolle, ob die Resultate des kleinen synthetischen Modells für die Umlegung und die beiden Kalibrierungsmethoden auch auf grosse reelle Modelle übertragbar sind.

Die Resultate zeigen, dass Erzeugung, Verteilung und Umlegung die Eingangsfehler nicht vergrössern, dass aber der Modal Split die Fehler mit einem Faktor von 3 bis 4 multiplizieren kann.

Die zwei Eingangsgrössen mit dem grössten Einfluss auf die Ausgangsfehler sind die Beziehungen der Quelle-Ziel-Matrix (= Wunschlinien) und die Linkzeiten. Die Eingangsfehler der Beziehungen beeinflussen die Ausgangsfehler der Linkbelastungen, die Eingangsfehler der Linkzeiten die Ausgangsfehler der Reisezeiten der Beziehungen.

Ein wesentliches Resultat dieser Studie betrifft die zwei Kalibrierungsmethoden. Die eine ist die klassische Entropie-Methode von Willumsen [Ortuzar, Willumsen]. Die andere, dessen additive Form, wurde vor bald 20 Jahren von [de Rham 87] entwickelt. Beide Methoden produzieren bestmögliche Schätzungen für die Linkbelastungen, die zweite jedoch mit einer wesentlich geringeren Verzerrung der resultierenden Distanzverteilung.

Ein Vergleich beider Methoden mit dem ARE Modell zeigte auf, dass die additive Methode Ausgangsfehler produziert, die für Linkbelastungen ca. 6 mal kleiner und für Reisezeiten ca. 3 mal kleiner sind als mit der klassischen Methode.

Damit endet eine alte Kontroverse zwischen Nutzer der Entropie-Methode, die die grosse Variabilität ihrer Resultate intuitiv ahnten und Nutzer der additiven Form, die nicht beweisen konnten, dass ihr Ansatz zuverlässiger war.

Die zwei wichtigsten Empfehlungen, um Ausgangsfehler zu vermindern, sind:

- prioritär die Eingangsfehler von Beziehungen und Linkzeiten reduzieren;
- die additive Form der Entropie anwenden, anstatt die klassische multiplikative Form.

Die Kombination beider Empfehlungen verhindert zudem, dass die Investition in bessere Quell-Ziel-Beziehungen und Linkzeiten wegen dem ungünstigeren Verhalten der klassischen Entropie-Methode zu nichte gemacht wird.

Dank der Fehlerfortpflanzung können neue Lösungen zum Problem des Qualitätsmonitoring von online Verkehrsmodellen gefunden werden. Die herkömmliche Methode der wiederholten Simulationen bedingt Mehraufwand in Programmierung für die Verwaltung der Input- und Output-Daten, mehrfache Versuche um die Signifikanz zu garantieren sowie Kenntnisse in multivariater Statistik für die Analyse der Resultate. Die Konsequenz ist, dass sie kaum angewandt wird.

Diese Nachteile verschwinden, sobald man die Regeln der Fehlerfortpflanzung explizit in die Modellierung integriert. Die Inputfehler der gemessenen Verkehrsmengen und Geschwindigkeiten pflanzen sich im Modell fort und produzieren Outputfehler der berechneten Verkehrsmengen und Geschwindigkeiten für jede Strecke im Netz.

Damit öffnen sich neue Möglichkeiten für ein effizientes Qualitätsmanagement von online Anwendungen. Die Schwachpunkte der Messinfrastruktur werden schnell identifiziert und behoben. Die Algorithmen der Kalibrierung werden gewählt um die numerischen Effekte der Fehlerfortpflanzung zu minimieren. Die Qualität der Verkehrslage, der Kurzfristprognosen, der Verkehrsinformation und der Reisezeiten kann somit 24 Stunden am Tag und 7 Tage die Woche überwacht und garantiert werden.

1. Introduction

1.1 What is the problem?

The problem is very simple: how do errors propagate through a transport model? There are historical reasons behind the fact that transport engineers did not give much attention to this subject in the past. Transport engineers have mainly civil engineering backgrounds and were not educated as physicists. Security is a central aspect in civil engineering. The engineers are used to calculate a mean value and to add security factors. In physics, precision is important. The value of a constant like the speed of light c need not be "on the safe side", but it is important to know its precision.

The error propagation poses a classic input-output problem: Data with some errors (or imprecision or uncertainties) are fed into a program. Some processes transform these inputs into outputs. The errors propagate through all steps of the process and are an integral part of the output results. The main transformation step in transport models is the search for optimal (or shortest) paths for the assignment. These algorithms have been studied over and over. On the other hand, and as far as we know, the behavior of errors during the transformation process has not been analyzed in detail.

We do not intend to reinvent the wheel in this study. The rules of error propagation are well known. What we want to do is:

- apply error propagation rules to transport models
- analyze the behavior of output errors as a function of input errors and model characteristics

Such a study is useless without numerical calculations. A tool must be used with which the explicit propagation of errors through transport models can be analyzed simply and efficiently.

Back in 1994, a student C. Luz of Prof. C. Hidber did a semester work under the supervision of C. de Rham. The student analyzed and programmed the error propagation for the path search and assignment in the programming language OBERON. In the mean time, C. de Rham translated this program in ADA and used it for different studies.

This ADA program, renamed "ERR_PROP", is the right tool for this study. It will be used mainly to try to find empirical rules or links connecting output errors to input errors and type of procedures.

The three original goals of the study were:

1) Acquire knowledge about error propagation in transport models. This goal was reached: the study shows how to treat error propagation in transport models and how to add error propagation capabilities to transport modeling software.

2) Estimate which input errors influence which output quantities and by how much. This goal was reached too: a total of 15 tests cases with a small example and 6 test cases with the Swiss ARE model give detailed answers to this question.

3) Produce a leaflet to help model users apply these findings in practice. The members of the steering group were not happy with this idea. They opposed the justified argument that a leaflet will never be referenced in any database and therefore never be accessible by any search engine. The idea of a leaflet was then abandoned in favor of trying to publish papers in specialized journals and/or congress proceedings.

1.2 State of the art, need for research

The report SVI/VSS 258, 46/90 „Fehlerrechnung und Sensitivitätsanalyse für Fragen der Luftreinhaltung“ addressed similar questions to the three goals above, but remained at a very general level and did not include any numerical example. Its utility for practical applications is very limited.

The paper from [de Jong] et al. sets objectives similar to those of our research: “… to develop a methodology to estimate the amount of uncertainty in forecasting for new infrastructure …” (citation from the introduction). The first part of the paper is an extensive literature review; the second one consists of recommendations for two Dutch forecasting models. The literature review concludes, “All methods encountered in the literature for quantifying the amount of input uncertainty use some form of repeated simulation”.

We deliberately did not want to follow this classical approach that has limits which are rapidly reached. If the complexity is increased to the level of realistic models, the results depend on too many factors and the saying “it’s too vague to be wrong” begins to be true.

Instead, we opted for a new approach where the error propagation is explicitly calculated in each single operation from the input of the raw data to the output of the results (mainly assigned traffic flow and travel times). Input and output are then fed into multiple regression analysis to see if and what relationship exists between them. The Dutch study confirms that our approach following the precise error propagation throughout all calculations is new in the field.

1.3 Applied method

1.3.1 Theoretical foundation and practical consequences

The study intends to treat the following themes

- Analysis and discussion of error propagation formulas
- How does error propagation affect the different operators?
- Priority for the choice of operators
- Strategies of improvement: which type of error has what influence?
- Cumulative effect of measurement errors and model complexity
- Methods to evaluate the effects of input errors on output results

1.3.2 The program ERR_PROP

The core of the program is based on a major capability of the computer language ADA, which is to “overload” operators. Any operator like “+” or “exp” can be overloaded to be applied to more complex data types than just simple integer or floating numbers. The application to error data consisting of a mean value and an error term is explained below in details.

ERR_PROP will be fed with input data having an error term as percentage of the value:

- Trip values 20% (➔ the input error is 20% of the trip value)
- Link times 10%
- Level of occupancy 15%
- Measured traffic 5%
- etc.

ERR_PROP will produce output data having an error term as percentage of the value. This error term is the result of the explicit application of error propagation rules throughout the whole model:

- Link flow 12.7% (➔ the resulting error is 12.7% of the link flow)
- Trip travel times 9.2%

- Trip values 17.5%
- etc.

Each combination of input errors can be viewed as a dot in a space of N dimensions. All dots together form some sort of cloud. The same argument holds for each combination of output errors. It is highly probable that the values in the output cloud are not independent of the values in the input cloud and that these dependencies can be detected with multiple regression analysis.

1.3.3 Application on the estimation of errors in traffic models

We apply the methods of error analysis of physical processes to transport modeling. We analyze separately the three main steps: Input, Process and Output.

The input data to transport models are:

- Network with nodes and links
- Valuation of nodes and links with distance, time and/or costs
- Trip matrix between zones
- Measured data (flow q and speed v) for calibration

The process consists of the four classical steps of transport modeling plus calibration

- Generation (step 1)
- Distribution (step 2)
- Modal split (step 3)
- Assignment (step 4)
- Calibration of the trip matrix with measured values of flow q and speed v

The output consists of either

- Values per link (traffic flow, times, tolls, generalized costs, etc.)
- Values per trip (calibrated trip value, travel times, tolls, generalized costs, etc.)

1.3.4 Should error terms be added to parameters of functions?

The question was raised if error terms should be added to the parameters of the functions used in the model, e.g.:

- Volume delay function: $T = T0\ (1 + \alpha\ (q/c)^{\beta})$ parameters α, β
- Distribution cost function $f\ (C_{ij}) = \alpha * C_{ij}^{\beta} * e^{-\gamma C_{ij}}$ parameters α, β, γ
- Modal split function $Pa = Ca^{-\alpha} / \Sigma(Ci^{-\alpha})$ parameter α

Error terms should only apply to physical quantities that are uncertain by their nature. There is no justification to add error terms to a function parameter.

A good example is Newton's law of gravity: $F = G * m1 * m2 / r^2$. The physical quantities G, m1, m2 and r have been measured and have therefore an error term that cannot vanish to zero. However, there is no justification whatsoever to add an error term to the exponent "2" of the distance r. Anyway, the propagation law for exponents will apply. The error term of the radius r will be multiplied by the factor "2" (see below: error propagation rules).

The case of parameters of functions used in the model is different because these parameters were not deducted analytically. They result from empirical estimations. It is probable that the authors of these estimations had some knowledge of the error terms, even if these are now lost. The application of propagation rules to the volume delay function is given in a special chapter below.

1.4. Test cases

There will be two test cases, a first one with a synthetic model and a second one with the Swiss traffic model from ARE.

The synthetic model will be used to simulate "laboratory" conditions exempt from uncontrollable external effects. The dimension will be some 20-30 zones, 400 nodes and 1000 links. This model will be fed with defined values of input errors. Possible dependencies between output errors and input errors will be analyzed with multiple regression.

The Swiss traffic model will be tested to see what consequences input errors have on a large-scale model. One reason for this choice is also that the ARE is directly interested in acquiring knowledge about the reliability of their modeling activity.

1.5. General work program

1.5.1 Step 1: literature, theory, update of ERR_PROP

- Search for references on Internet
- List of the input variables of transport models
- Estimation of probable error ranges for these input values
- Search for methods to estimate and reduce these error ranges
- List of the output variables of transport models

1.5.2 Step 2: practical tests

Update of ERR_PROP to suit to this study

Work with the synthetic model:

- Generation, distribution, modal split
- Assignment and calibration of trip matrices
- Definition of a reasonable maximal error for each input variable
- Run tests with combinations of different values of the input errors
- Extract the input and output error data in a suitable form for regression calculation
- Search for relations between input and output errors

Work with the Swiss traffic model from ARE

- Assignment and calibration of the Swiss trip matrices
- Reduced number of test cases
- Special tests for questions of road pricing (to be discussed with ARE)

1.5.3 Step 3: synthesis, report

- Synthesis of the results, conclusions
- Effect of input errors on output errors
- Methods to reduce the errors in transport models, pitfalls to avoid
- Recommendations for the definition of requirements for transport models
- Research report

1.6. Expected results and utility

- Answer to the question: **Which input error influences which output error and by how much?**
- Contribution to better understanding the mechanisms of error propagation.
- Practical tools like tables, regression equations, etc. to help the user of transport modeling to estimate the relations between input and output errors.
- At least one paper resuming the research findings will be submitted to:
  - Journals like Strasse und Verkehr or Strassenverkehrstechnik
  - Conferences like European or World Congress on ITS

2 Basics of Error Analysis

This chapter is largely inspired from [Taylor].

2.1 Definition of « error »

Error is defined here as the uncertainty in measurement. No measurement, however carefully made, can be completely free of uncertainty. In science, the word error does not carry the usual connotation of terms like mistake or blunder. Error is simply the inevitable uncertainty that appears in all measurements. It is impossible to eliminate errors, even by being very careful or investing in apparatus that are more precise.

As these uncertainties are inevitable, the only things one can do are to:

a) try to keep errors of input data as small as possible,

b) estimate the consequence of these input errors on calculated outputs.

Step a) is under the responsibility of the engineer building his model.

Step b) is the subject of this research.

2.2 Measurement and use of uncertainties

The simplest way to state the result of a measurement is to give a best estimate of the value and a confidence interval.

- best estimate x_{best}
- confidence interval from $x_{best} – dx$ to $x_{best} + dx$
- measured value $X = x_{best} \pm dx$

2.3 Significant figures

Two rules should be applied when stating measured values and their precision. The derivation and justification can be found in [Taylor].

Rule 1 applies when stating uncertainties:

Experimental uncertainties should be rounded to one significant figure

This can be illustrated with the acceleration of gravity. To state that $g = 9.82 \pm 0.02385$ m/s^2 is absurd, whereas $g = 9.82 \pm 0.02$ m/s^2 is correct.

Rule 2 applies when stating answers:

The last significant figure of the estimate should be of the same order of magnitude as the uncertainty.

Here again, to state that a speed $v = 68.241536 \pm 4$ m/s makes no sense, the right statement would be $v = 68 \pm 4$ m/s.

2.4. Propagation of uncertainties

2.4.1 The general case

The derivation of this formula can be found in [Taylor] page 73 ff.

If q is any function of several variables x, … ,z then

$q = f(x, \dots ,z)$

$dq = SQRT\{ ([\delta q/\delta x]*dx)^2 + \dots + ([\delta q/\delta z]*dz)^2 \}$

where δq/δx, …, δq/δz are the partial derivatives of q.

All subsequent formulas can be derived from this one.

2.4.2 Sums, differences, products and quotients

The following rules apply for sums and differences:

Assume several quantities x, ….. ,w are measured with uncertainties dx, ….. , dw.

If q is computed:

$q = (x + \dots + z) – (u + \dots + w)$

the partial derivatives will all be equal to one:

$\delta q/\delta x = 1, \delta q/\delta y = 1$, etc.

and the total uncertainty dq will be:

$dq = SQRT\{dx^2 + \dots dz^2 + du^2 + \dots + dw^2\}$

Due to the triangle inequality[1], the total uncertainty is never larger than the sum of the absolute values of the single uncertainties

$dq <= dx + \dots dz + du + \dots + dw$, upper limit

The absolute errors always add. It does not depend on whether the estimates are added or subtracted.

For products and quotients, one uses the fractional uncertainty.

If X is defined as

$X = x_{best} \pm dx$

then the fractional uncertainty in x is defined as

$dx / | x_{best} |$

or shorter

$dx / | x |$.

The value of X can be expressed as

$X = x (1 \pm (dx /| x |))$.

Assume several quantities x, … ,w with small uncertainties dx, … ,dw.

If q is computed:

$q = (x * \dots * z) / (u * \dots * w)$

then the fractional uncertainty of q will be

$dq/ |q| = SQRT\{ (dx/ |x|)^2 + \dots + (dz/ |z|)^2 + \dots + (du/ |u|)^2 + \dots + (dw/ |w|)^2 \}$

here again, because of the triangle inequality, the total fractional uncertainty is never larger than the sum of the single fractional uncertainties

$dq/ |q| <= dx/ |x| + \dots + dz/ |z| + \dots + du/ |u| + \dots + dw/ |w|$, upper limit

The relative errors always add. It does not depend on whether the estimates are multiplied or divided.

[1] triangle inequality: if $y^2 = \Sigma x_i^2$ then $y <= \Sigma |x_i|$ (always, in Euclidean geometry)

2.4.3 Other functions

Exact number multiplied by a measured quantity q = B * x:

$q = B * x$

$dq = |B| * dx$

Exact number divided by a measured quantity q = B / x:

$q = B / x$

$dq = |B| * (dx / x^2)$

Measured quantity to a power $q = x^n$:

$q = x^n$

$dq = |n| * |x^{n-1}| * dx$

$dq = |n| * (|q| / |x|) * dx$

$dq / |q| = (|n| / |x|) * dx$

Measured quantity in the exponent $q = a^x$:

$q = a^x$

$dq = a^x \ln(a) * dx$

2.5 Analytical example with the volume-delay function

The volume-delay function with parameters α, β, traffic flow q and capacity c is a good candidate to illustrate the application of error propagation rules.

$T = T0\ f(\alpha, \beta, q, c)$

where

$F = f(\alpha, \beta, q, c) = 1 + \alpha * (q/c)^\beta$

The road capacity c is assumed to be error-free, but not parameters α, β and traffic flow q. From the general case above, we have:

$$dF = SQRT\{ ([\delta F/\delta\alpha]*d\alpha)^2 + ([\delta F/\delta\beta]*d\beta)^2 + ([\delta F/\delta q]*dq)^2 \}$$

$$\delta F/\delta\alpha = (q/c)^\beta \quad = \alpha*(q/c)^\beta \quad * (1/\alpha)$$

$$\delta F/\delta\beta = \alpha*(q/c)^\beta * \ln(q/c) \quad = \alpha*(q/c)^\beta \quad * \ln(q/c)$$

$$\delta F/\delta q = \alpha*\beta * (q/c)^{\beta-1} \quad = \alpha*(q/c)^\beta \quad * (\beta / (q/c))$$

and $$dF = \alpha*(q/c)^{\beta} * SQRT\{ ((1/\alpha)*d\alpha)^2 + (\ln(q/c)*d\beta)^2 + ((\beta*c/q)*dq)^2 \}$$

The approximation is easy for flow q near capacity c, or q/c ≈ 1. With the normal values of α = 0.15 and β = 4, we have $\alpha*(q/c)^\beta \approx 0.15$, (1/α) ≈ 7, ln(q/c) ≈ 0 and (βc/q) ≈ 4. The result for q/c ≈ 1 is then:

$$dF \approx 0.15 * SQRT\{ (7*d\alpha)^2 + (4*dq)^2 \}$$

As the value of dα is negligible compared to dq, the expression simplifies to:

$dF \approx 0.15 * 2 * dq$ or **$dF \approx 0.3 * dq$** **for $q/c \approx 1$**

Another example with q/c ≈ ½ leads to $\alpha*(q/c)^\beta \approx 0.01$, (1/α) ≈ 7, ln(q/c) ≈ -0.7 and (βc/q) = 2. The total error dF is then:

$$dF \approx 0.01 * SQRT\{ (7*d\alpha)^2 + (-0.7*d\beta)^2 + (2*dq)^2 \}$$

Again, as dα and dβ are negligible compared to dq, the result will be:

$dF \approx 0.01*SQRT(2)*dq$ or **$dF \approx 0.014 * dq$** **for $q/c \approx ½$**

Conclusion: The error due to the volume-delay function is roughly equal to 0.3 * dq for flow near capacity and decreases sharply with decreasing flow values.

2.6 Implementation with function overloading

The software language used to implement the error propagation is ADA. ADA has the capability to overload operators. That means that the operator will behave according to the data types submitted for an operation. Here is an example of overloading for the addition operation c = a + b. Both values a and b have uncertainties da and db. From above, we know that the following rules apply:

```
c = a + b                  -- addition of the best estimates
dc = sqrt(da^2 + db^2)     -- error propagation
```

ADA allows to overload the operator "+" by error propagation rules. To do this, we define a new data type T_XD with two components X (best estimate) and D (error).

```
type T_XD is record X, D: float;
```

The function which adds the two values A and B of type T_XD and returns a result (also of type T_XD) looks like:

```
function „+" (A, B: in T_XD; C: out T_XD) is
    C.X := A.X + B.X                       -- addition of the best estimates
    C.D:= sqrt((A.D*A.D) + (B.D*B.D))      -- error propagation
    return C;                              -- returns the result
```

Such functions are defined for all operators (+, -, *, /, exponent, power, etc.), which appear in the program. The rest of the software can be written without bothering with these details. The program recognizes automatically which function of what type is applicable in each case.

This overloading capability is very convenient to import procedures from other sources without having to modify them. The software ERR_PROP was built according to these principles.

All input, intermediate and output variables used in connection with error propagation must be of type T_DX. To ensure this consistency throughout the program is easy by implementing only overloaded functions which return a type T_DX.

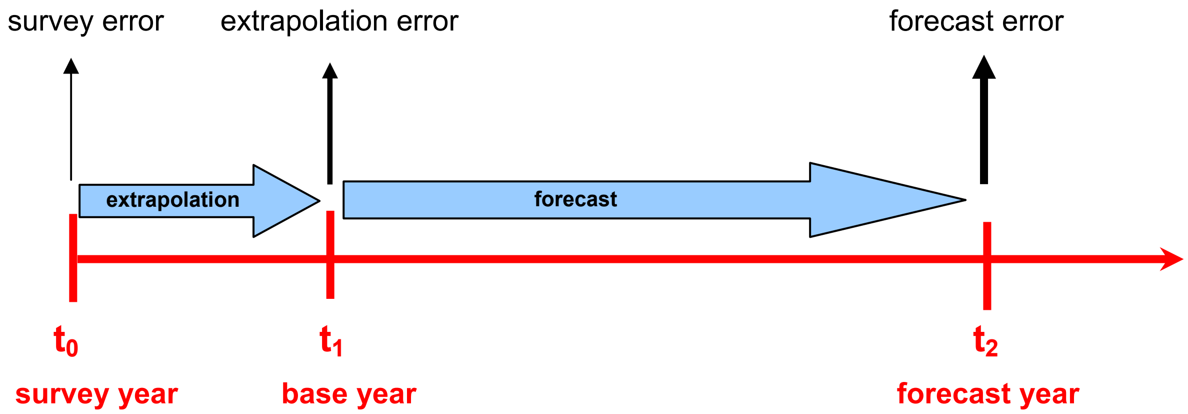
3 Propagation in Transport Models

3.1 Input data: estimation of error ranges

The following estimations are based on the experience of the research team in collecting data and building transport models. It seems to be very difficult to find literature about this subject, even after extensive search on Internet.

3.1.1 Errors of zone data

Zone data such as number of inhabitants, work places, occupation, etc. are usually available from communities, cantons or the federal government. These data are regularly updated every 5 to 10 years (national population survey, industry surveys, etc.).



The different types of data are seldom available from the same period. The use for modeling implies that the data must first be extrapolated to some base year and then forecasted for future periods. Errors can appear during the survey (t_0), the extrapolation for the base year (t_1) and the extrapolation errors for forecast (t_2).

Survey errors are very small and lie in the range +/- 1%, whereas extrapolations are naturally error prone. The horizon is normally 10 years ahead, forecasted with linear extrapolation. Error ranges of +/- 20% are typically assumed.

3.1.2 Errors of trip data

Trips are normally generated from inhabitants and work places and their specific production of trips per day. According to [Dietrich], accepted values for specific productions are the following:

- per inhabitant: 1.5 to 2.5 trips/day
- per work place: 3.0 to 8.0 trips/day
- per customer intensive services: 5 to 13 trips/day

The error ranges vary widely between types of traffic planning project, but can be made available from the data source. The error margin should not be greater than +/- 20%.

Data used for distribution like commuter statistics, micro census and license plate surveys also have different types of errors:

- survey errors, e.g. the micro census is only a representative sample
- erroneous and/or old data
- uncertainties during forecasts

The variety of error sources is particularly high with license plate surveys:

- errors when extrapolating from the survey time interval to a mean hourly value

- input of a wrong plate number
- the identity of the last digits does not mean that the full number is the same
- errors during the data analysis

Error ranges of +/- 10% are realistic for license plate surveys.

3.1.3 Errors of distances and times

Distances between nodes are normally based on highly accurate coordinate data. As these errors will always be about an order of magnitude smaller than all other errors considered in the model, distances will be taken as exact values with no errors.

The accuracy of speed depends on the load of the network. For lightly loaded links, one can assume the signalized maximum speed without error. The case is more complex for heavily loaded links, because of the following circular dependencies: higher link flow ➔ lower link speed ➔ higher link time ➔ lower attractivity of the path from origin to destination through this link ➔ shift of some trips or part of trips to more attractive paths ➔ lower link flow ➔ etc.

All speed errors are converted to time errors. This is to be coherent with the travel time error that depends only on the addition rules for link times on the path from origin to destination.

Node times were not considered in this study. This is not because their importance is not recognized, especially for urban models. The reason is that if the modeling of node times has to be realistic, that is with priority roads, pre-selections, left turns, etc. the complexity and need for precise data rises sharply. This is beyond the scope of this study.

3.1.4 Errors of measured data

The main sources of traffic measurement are induction loops, video cameras and floating car data. The accuracy of induction loops and video cameras can be taken from the manufacturer's specifications and [BAST]. These are normally:

- traffic flow q of short vehicles (cars)	$dq/q < 5$ %
- traffic flow q of long vehicles (trucks)	$dq/q < 3$ %
- speed $v < 100$ km/h	$dv < 3$ km/h
- speed $v > 100$ km/h	$dv/v < 3\%$

The accuracy of floating car data, **FCD,** depends on the traffic intensity q[vhc/km], the proportion p% of equipped vehicles, the time interval dt and the number N of required observations. Analyses of these dependencies can be found in [de Rham 03].

3.2 Four step model, consequences on errors

The model process consists of the four classical steps of transport modeling plus calibration

1 Generation
2 Distribution
3 Modal split
4 Assignment (based on calculation of paths from origins to destinations)
5 Calibration of the trip matrix with measured values of flow q and speed v where:
    5.1 Calibration with a multiplicative method
    5.2 Calibration with an additive method

3.2.1 Generation, step 1

The generation of production P and attraction A [# of trips / unit of time] per zone is calculated with:

$P(i) = \Sigma (ZD(i,k) * M(k))$

$A(i) = \Sigma (ZD(i,k) * M(k))$

Where

i = index of zone

i_max = number of zones

k = index of zone data type (1 = population, 2 = workplaces, 3 = …)

k_max = number of zone data types

ZD(i,k) = value of zone data type k for zone i

M(k) = mobility factor for zone data type k

To get a first estimation of the consequences of error propagation on the generation step, one can make some simplifying assumptions:

zone data Z = z + dz where z and dz have the same values for all zones

mob. factors M = m + dm where m and dm have the same values for all factors

The single product P = Z * M = p + dp will have the components

$p = z * m$

$dp = p * SQRT((dz/z)^2 + (dm/m)^2)$

The production per zone is the sum of the products for each zone data

P = prod + dprod

Where

$prod = \Sigma p = \Sigma(z * m)$

$dprod = SQRT(\Sigma(dp)^2)$

$dprod = SQRT(k\_max * (dp)^2)$

$dprod = dp * SQRT(k\_max) = p * SQRT((dz/z)^2 + (dm/m)^2) * SQRT(k\_max)$

The production total for all zones is the sum of the production of each zone

PT = prodt + dprodt

Where

$prodt = \Sigma prod = i\_max * prod$

$dprodt = SQRT(\Sigma(dprod)^2)$

$dprodt = SQRT(i\_max * (dprod)^2)$

$dprodt = dprod * SQRT(i\_max)$

$dprodt = p * SQRT((dz/z)^2 + (dm/m)^2) * SQRT(k\_max) * SQRT(i\_max)$

If the numerical values of the product p = z * m are of the same order, one can assume that prod = p * k_max and simplify the expressions for the relative error dprod/prod per zone:

$dprod/prod = (p * SQRT((dz/z)^2 + (dm/m)^2) * SQRT(k\_max)) / (p * k\_max)$

➔ **$dprod/prod = SQRT((dz/z)^2 + (dm/m)^2) / SQRT(k\_max)$**

Numerical example with 25 zones, each with 3 types of zone data and uniform errors of 10% for zone data and mobility factors. What is the relative error of the production per zone and of the total production?

zones: i_max = 25

data types: k_max = 3

zone data: z = 1000 dz = 100 dz/z = 0.1, for all 3 types

mob. factor: m = 1 dm = 0.1 dm/m = 0.1, for all 3 types

a) error of a single product

p = z * m = 1000 * 1 = 1000

$dp = p * SQRT((dz/z)^2 + (dm/m)^2) = 1000 * SQRT((0.1)^2 + (0.1)^2) = 141$

dp/p = 141 / 1000 = 0.141 = 14%

b) relative error of the production per zone

prod = Σp = 3000

dprod = SQRT(k_max) * dp = SQRT(3) * 141.2 = 245

dprod/prod = 245 / 3000 = 0.08 = 8%

c) relative error of the production total of the 25 zones

prodt = Σprod = 75000

$dprodt = p * SQRT((dz/z)^2 + (dm/m)^2) * SQRT(k\_max) * SQRT(i\_max)$

= 1000 * SQRT(2) * 0.1 * SQRT(3) * SQRT(25) = 1225

dprodt/prodt = 1225 / 75000 = 0.016 = 2%

Result of program ERR_PROP (example):

LIST OF ZONES: PRODUCTION, ATTRACTION

zone	mean	PROD estim	in %	PROD input	in %	ATTR estim	in %	ATTR input	in %
total:	PKW	0± 0%	0.00%	75000.0± 2%	100.00%	0± 0%	0.00%	75000.0± 2%	100.00%
1001	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
1006	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
1011	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
1016	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
1021	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
6001	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
6006	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
6011	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
6016	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
6021	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
11001	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
11006	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
11011	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
11016	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
11021	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
16001	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
16006	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
16011	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
16016	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
16021	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
21001	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
21006	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
21011	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
21016	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%
21021	PKW	0± 0%	0.00%	3000.0± 8%	4.00%	0± 0%	0.00%	3000.0± 8%	4.00%

This output of step 1 can be fed as input to the next step 2.

3.2.2 Distribution, step 2

The row and column sums of trips (=margins of the trip matrix) will be distributed with the following iterative algorithm:

$T_{ij} = Q_i Z_j q_i z_j * f(C_{ij})$

Q_i = PROD (i) $\quad$ Zj = ATTR (j)

$q_i = 1 / (\sum Z_j z_j * f(C_{ij}))$ $\quad$ $z_j = 1 / (\sum Q_i q_i * f(C_{ij}))$

C_{ij} = generalized costs from zone i to zone j

$f(C_{ij}) = \alpha * C_{ij}^{\beta} * e^{-\gamma C_{ij}}$ $\quad$ α, β, γ = parameters

Example: the combination of α = 1, β = -2, γ = 0 corresponds to the gravity model

These equations are too complex to analyze the error propagation analytically. This will also be the case for the other methods described below. This is where the program ERR_PROP will be of great help: it does all the work of propagation according to the exact rules through all the thousands steps of calculation.

Here is a numerical test with the following errors:

- times $\quad$ t ± dt = t (1 ± 10%) $\quad$ (again: chosen arbitrarily to 10%)
- production $\quad$ p ± dp = p (1 ± 8%) $\quad$ (result of step 1)

Results of program ERR_PROP (example):

LIST OF ZONES: PRODUCTION, ATTRACTION

zone	mean	PROD estim	in %	PROD input	in %	ATTR estim	in %	ATTR input	in %
total:	PKW	75000.0± 2%	100.00%	75000.0± 2%	100.00%	75000.0± 2%	100.00%	75000.0± 2%	100.00%
1001	PKW	3000.0±11%	100.00%	3000.0± 8%	4.00%	3000.0± 9%	100.00%	3000.0± 8%	4.00%
1006	PKW	3000.0±10%	100.00%	3000.0± 8%	4.00%	3000.0± 9%	100.00%	3000.0± 8%	4.00%
1011	PKW	3000.0± 8%	100.00%	3000.0± 8%	4.00%	3000.0±10%	100.00%	3000.0± 8%	4.00%
1016	PKW	3000.0±11%	100.00%	3000.0± 8%	4.00%	3000.0±12%	100.00%	3000.0± 8%	4.00%
1021	PKW	3000.0±14%	100.00%	3000.0± 8%	4.00%	3000.0±11%	100.00%	3000.0± 8%	4.00%
6001	PKW	3000.0± 8%	100.00%	3000.0± 8%	4.00%	3000.0± 9%	100.00%	3000.0± 8%	4.00%
6006	PKW	3000.0± 8%	100.00%	3000.0± 8%	4.00%	3000.0± 9%	100.00%	3000.0± 8%	4.00%
6011	PKW	3000.0± 7%	100.00%	3000.0± 8%	4.00%	3000.0± 8%	100.00%	3000.0± 8%	4.00%
6016	PKW	3000.0± 7%	100.00%	3000.0± 8%	4.00%	3000.0± 8%	100.00%	3000.0± 8%	4.00%
6021	PKW	3000.0± 9%	100.00%	3000.0± 8%	4.00%	3000.0±11%	100.00%	3000.0± 8%	4.00%
11001	PKW	3000.0± 9%	100.00%	3000.0± 8%	4.00%	3000.0± 8%	100.00%	3000.0± 8%	4.00%
11006	PKW	3000.0± 7%	100.00%	3000.0± 8%	4.00%	3000.0± 7%	100.00%	3000.0± 8%	4.00%
11011	PKW	3000.0± 7%	100.00%	3000.0± 8%	4.00%	3000.0± 7%	100.00%	3000.0± 8%	4.00%
11016	PKW	3000.0± 7%	100.00%	3000.0± 8%	4.00%	3000.0± 7%	100.00%	3000.0± 8%	4.00%
11021	PKW	3000.0± 9%	100.00%	3000.0± 8%	4.00%	3000.0± 9%	100.00%	3000.0± 8%	4.00%
16001	PKW	3000.0±11%	100.00%	3000.0± 8%	4.00%	3000.0±12%	100.00%	3000.0± 8%	4.00%
16006	PKW	3000.0± 7%	100.00%	3000.0± 8%	4.00%	3000.0± 9%	100.00%	3000.0± 8%	4.00%
16011	PKW	3000.0± 7%	100.00%	3000.0± 8%	4.00%	3000.0± 7%	100.00%	3000.0± 8%	4.00%
16016	PKW	3000.0± 8%	100.00%	3000.0± 8%	4.00%	3000.0± 7%	100.00%	3000.0± 8%	4.00%
16021	PKW	3000.0± 8%	100.00%	3000.0± 8%	4.00%	3000.0± 8%	100.00%	3000.0± 8%	4.00%
21001	PKW	3000.0±16%	100.00%	3000.0± 8%	4.00%	3000.0±13%	100.00%	3000.0± 8%	4.00%
21006	PKW	3000.0±10%	100.00%	3000.0± 8%	4.00%	3000.0±13%	100.00%	3000.0± 8%	4.00%
21011	PKW	3000.0±10%	100.00%	3000.0± 8%	4.00%	3000.0± 8%	100.00%	3000.0± 8%	4.00%
21016	PKW	3000.0± 9%	100.00%	3000.0± 8%	4.00%	3000.0± 9%	100.00%	3000.0± 8%	4.00%
21021	PKW	3000.0± 8%	100.00%	3000.0± 8%	4.00%	3000.0±10%	100.00%	3000.0± 8%	4.00%

3.2.3 Modal Split, step 3

The Modal Split is applied with the classic logit function to the different paths i from origin O to destination D [Schnabel/Lohse]. Each path i has generalized costs Ci. The part of traffic following path a is equal to:

$$Pa = Ca^{-\alpha} / \Sigma(Ci^{-\alpha})$$

where α is a parameter which can vary theoretically from 0 to ∞. With α = 0, all paths will be equivalent, with α = ∞, the best path will have a probability of 100%. Normal values of α vary from about 6 for urban to about 10 for interurban networks. The value α will be set to 6, 8 and 10 for the tests and used as an independent variable in the regression equation.

3.2.4 Assignment, step 4

3.2.4.1 Successive assignment method

The basic assignment procedure consists of two steps, first to calculate best paths between origins and destinations, second to add the values of the trip matrix to the links belonging to each path. Errors will propagate during the path-finding step as well as the loading step.

Most models apply some sort of successive assignment also called incremental loading. The matrix is cut into N slices which are successively assigned to the network. Link costs and best paths are recalculated between each step. All these methods only add traffic to the network during successive iterations.

```
start with empty network
for K in 1..N loop
    calculate generalized costs of all links
    calculate best paths between all pairs of zones
    assign slice K of the trip matrix on network
end loop
```

Even if this adding is done with many iterations, successive assignment cannot guarantee convergence towards the so-called user equilibrium (UE) according to the Wardrop principle (see next paragraph). The reason is that for heavily loaded networks, the trip costs of the first assigned slices may be too cheap. In the end, all others slow down all users and it is quite possible that users of the first assigned slices would have chosen a different path.

3.2.4.2 Wardrop assignment with incremental reload

The Wardrop principle asserts that no user can decrease his generalized costs (= monetary costs + time weighted by the value of time) by taking another decision. This corresponds to the user equilibrium UE. The UE does not necessarily equal the system equilibrium or SE, which is the global optimum. The art of traffic engineers consists also in elaborating systems where the selfish behavior of users leads naturally towards SE. UE is near to the SE when (small) personal advantages cannot be any more exchanged for (large) welfare losses to all others.

The Wardrop principle is simple to formulate but not so simple to implement. Some models use mathematical programming, but their need for smooth (differentiable) functions renders them useless as soon as public transport with timetables is involved. The successive assignment does not guarantee a solution either, as paths chosen for their optimality during first slices may not stay optimal when all slices have been assigned. In short: successive assignment does not guarantee the convergence towards a Wardrop solution.

One way to solve this convergence problem is to apply “lateral thinking”: **do the assignment twice**, the second time by just removing matrix slices with “old” costs and re-assigning them with actual costs.

```
start with empty network
for K in 1..2*N loop
    if K > N then DEDUCT slice (K mod N)+1 from network; end if
    calculate generalized costs of all links
    calculate best paths between all pairs of zones
    assign slice (K mod N)+1 on network
end loop
```

In steps K = N+1 to K = 2*N, all slices are re-assigned to the network preloaded with N-1 precedent slices. It guarantees the UE with a precision of at least (100/N)% because now every slice has been assigned with costs due to the N-1 precedent ones. If a precision of say 5% is required, N = 20 will suffice. Technically, the assignment results must be kept individually for each of the N steps. This is easy to implement with an array of N cells for each link.

The method can be used with a wide range of applications. It works well with networks which offer many parallel routes, intermodal applications with a mix of individual transport, public transport with timetables, park + ride systems, etc.

3.2.5 Calibration

If measured values of flow are available, the result of the assignment will never fit these measured values exactly. It is therefore necessary to calibrate the matrix to adjust the assigned values with the measured ones. There are many possibilities of calibration. Due to contracts from the Stab Gesamtverkehrsfragen under the direction of Prof. C. Hidber back in 1986, C. de Rham studied different methods, especially the Entropy maximizing method and developed it's additive counterpart which is still widely in use. See also [de Rham 04].

3.2.5.1 Entropy method (multiplicative)

The Entropy maximization [FGSV 365] is mathematically very elegant but has the tendency to "over-adjust" trips which go through a small number of sensors. This distorts the trip length distribution and the structure of the matrix. If the links have a majority of measured values which are higher than the assigned ones, the method will add too much traffic to short trips and not enough to long ones. The Entropy method can be adjusted to correct for this effect, by adding correction factors depending on the number of measured cross sections encountered from origin to destination.

```
initialise measured link flow value ma for all links a
Initialise link calibration factors xa = 1.0 for all links a
Initialise trip calibration factors cij = 1.0 for all trips tij
loop
        assign matrix tij to produce link flow qa
        if convergence or max. number of iterations is reached: exit
        if ma > 0 then xa(n) = xa(n-1) * (ma / qa)
        sxij = product of xa on path from i to j
        nij = number of links on path from i to j
        cij(n)= cij(n-1) * sxij ^(1/nij)
        tij(n) = tij(n-1) * cij(n)
end loop
```

3.2.5.2 Entropy method (additive)

The original Entropy method is multiplicative with the disadvantage of over or under estimating short trips. It was then logical to search for it's additive counterpart, which would induce less distortion. This method was developed by C. de Rham thanks to contracts with the Stab für Gesamtverkehrsfragen [de Rham 86] and [de Rham 87]. At the level of links, the results are about as good as for the original Entropy method, but with much less distortion of the trip values. Both methods are roughly equivalent from the point of view of memory allocation and computing time.

```
initialise measured link flow value ma for all links a
Initialise link calibration factors xa = 0.0 for all links a
Initialise trip calibration factors cij = 1.0 for all trips tij
loop
        assign matrix tij to produce link flow qa
        if convergence or max. number of iteration is reached: exit
        if ma > 0 then xa(n) = (qa-ma) / ma
        sxij = sum of xa on path from i to j
        nij = number of links on path from i to j
        cij(n)= cij(n-1) – (sxij / nij)
        tij(n) = tij(n-1) * cij(n)
end loop
```

4 Tests with a Synthetic Model

4.1 Model setup

The network is a simple grid with cell dimensions increasing from center to the edges. There are 21 x 21 = 441 nodes and 5 x 5 = 25 zones (gray labels). The light gray roads correspond to the “actual state” S0 used in all examples. The light plus the dark gray roads correspond to a “future state” S1 that is compared to state S0 in tests T14 and T15.

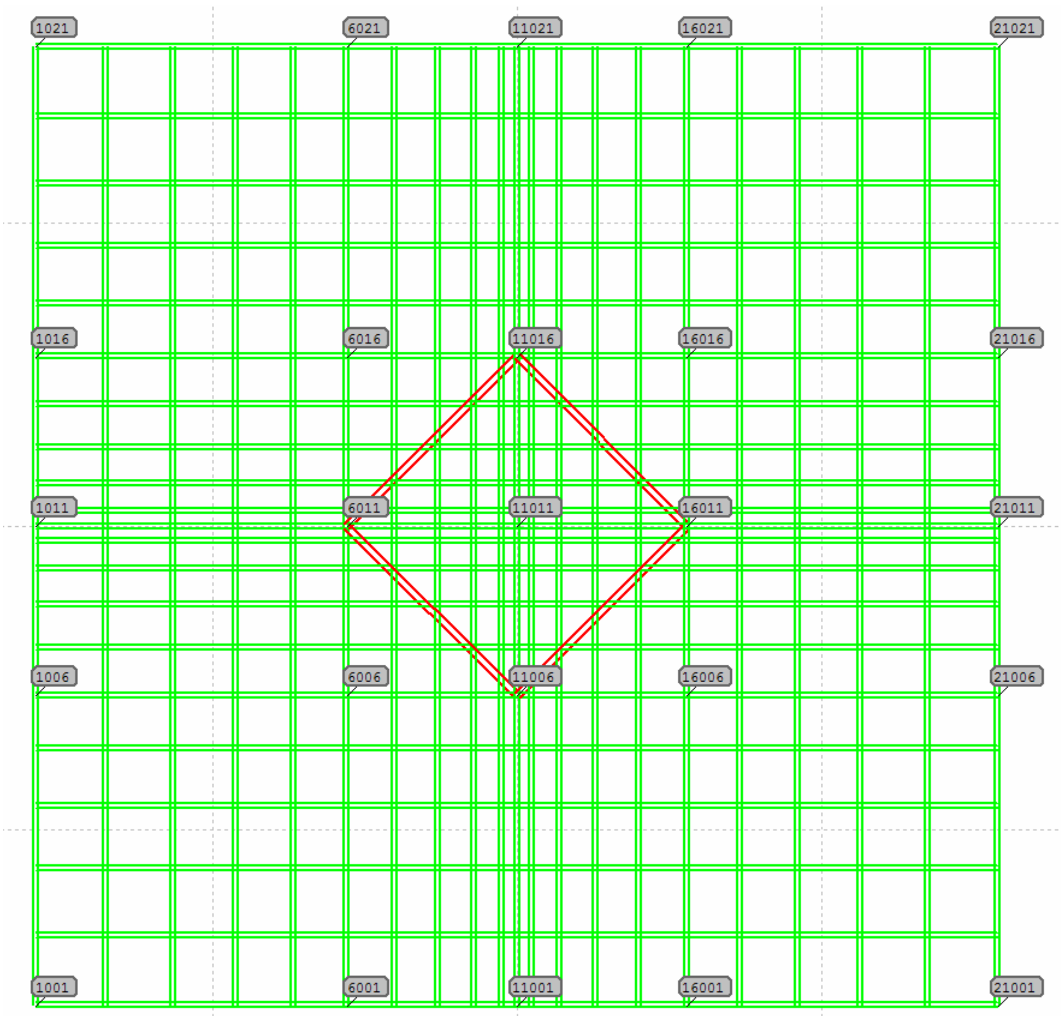


Fig 1: synthetic network

There are 3 types of zone data: ZDA1, ZDA2 and ZDA3 which could be population, workplaces, or any other value contributing to the production or attraction of trips. The three types of data have the following (fully arbitrary) characteristics

ident.	mean	sigma	mob. factor	resulting trips
ZDA1	1000	100	1	1000
ZDA2	100	10	10	1000
ZDA3	10	1	100	1000

It is easy to see that the mean production and attraction will be of the order of 3000 trips per zone and a total of about 25 * 3000 = 75’000 trips for the whole matrix.

4.2 Test runs

Each test is run with a new set of independent input error variables. Each dependent output error is analyzed as a function of all independent ones. The choice and combination of independent and dependent variables are summarized below for each test case:

Test#	indep. error	dep. error	min / step / max error range (%)
T01, generation			
	E_MOBI, mobility factors		0 / 2 / 20%
	E_ZDAT, zone data		0 / 2 / 20%
		E_PROD, production	
		E_ATTR, attraction	
T02, distribution			
	E_PROD, zone production		0 / 5 / 20%
	E_ATTR, zone attraction		0 / 5 / 20%
	E_TIME, link time		0 / 2 / 20%
		E_TRIP, trip value	
T03, modal split			
	E_TIME, link time		0 / 2 / 20%
	E_TRIP, trip value		0 / 2 / 20%
	EXPO, exponent		-6, -8, -10
		E_TR_A, trip value mode A	
		E_TR_B, trip value mode B	
T04/T05, assignment			
T04 analysis by link			
	E_TIME, link time		0 / 2 / 20%
	E_TRIP, trip value		0 / 2 / 20%
	NTRI, #trips on link		from model
		E_QINI, link flow, initial	
T05 analysis by O / D relation			
	E_TIME, link time		0 / 2 / 20%
	E_TRIP, trip value		0 / 2 / 20%
	ARCS, #arcs on path		from model
		E_TRAV, trip travel time	

NTRI and ARCS were introduced to check if the number of trips using a link and/or the number of arcs used by a trip had any influence on output errors. After many attempts and the conclusion was that there is no relation at all, they were abandoned.

Test#	indep. error	dep. error	min / step / max error range (%)
T06/T07, assignment with multiplicative calibration			
T06 analysis by link			
	E_TIME, link time		0 / 5 / 20%
	E_TRIP, trip value		0 / 5 / 20%
	E_QMES, link flow, counts		0 / 5 / 20%
	PMES, % counted links		0 / 5 / 20%
		E_QCAL, link flow, calibr.	
T07 analysis by O / D relation			
	E_TIME, link time		0 / 5 / 20%
	E_TRIP, trip value		0 / 5 / 20%
	E_QMES, link flow, counts		0 / 5 / 20%
	PMES, % counted links		0 / 5 / 20%
		E_TRIC, trip value calibr.	

Test#	indep. error	dep. error	min / step / max error range (%)
T08/T09, assignment with additive calibration			
T08 analysis by link			
	E_TIME, link time		0 / 5 / 20%
	E_TRIP, trip value		0 / 5 / 20%
	E_QMES, link flow, counts		0 / 5 / 20%
	PMES, % counted links		0 / 5 / 20%
		E_QCAL, link flow, calibr.	
T09 analysis by O / D relation			
	E_TIME, link time		0 / 5 / 20%
	E_TRIP, trip value		0 / 5 / 20%
	E_QMES, link flow, counts		0 / 5 / 20%
	PMES, % counted links		0 / 5 / 20%
		E_TRIC, trip value calibr.	
T10/T11, generation + distribution + modal split + assignment			
T10 analysis by link			
	E_MOBI, mobility factors		0 / 5 / 20%
	E_ZDAT, zone data		0 / 5 / 20%
	E_TIME, link time		0 / 5 / 20%
	EXPO, exponent		-6, -8, -10
		E_QINI, link flow, initial	
T11 analysis by O / D relation			
	E_MOBI, mobility factors		0 / 5 / 20%
	E_ZDAT, zone data		0 / 5 / 20%
	E_TIME, link time		0 / 5 / 20%
	EXPO, exponent		-6, -8, -10
		E_TRAV, trip travel time	
T12/T13, equilibrium assignment			
T12 analysis by link			
	E_TIME, link time		5 / 5 / 20%
	E_TRIP, trip value		5 / 5 / 20%
	ALFA, BPR function		0.65 / 0.25 / 1.40
	BETA, BPR function		2.0 / 1.0 / 5.0
		E_QINI, link flow, equilibrium	
T13 analysis by O / D relation			
	E_TIME, link time		5 / 5 / 20%
	E_TRIP, trip value		5 / 5 / 20%
	ALFA, BPR function		0.65 / 0.25 / 1.40
	BETA, BPR function		2.0 / 1.0 / 5.0
		E_TRAV, trip travel time, equilibrium	
T14/T15, comparison of two states S0 (actual) and S1 (future)			
T14 analysis by link			
	E_TIME, link time		2 / 2 / 20%
	E_TRIP, trip value		2 / 2 / 20%
		E_QDEL, link flow, difference	
T15 analysis by O / D relation			
	E_TIME, link time		2 / 2 / 20%
	E_TRIP, trip value		2 / 2 / 20%
		E_TDEL, trip travel time, difference	

4.3 Results

All the calculations were done with EXCEL. As the intercept of the regression equation is not significant in most cases and very small in the others, it is systematically forced to be zero. It also makes the regression equations easier to interpret and to apply.

4.3.1 Generation

T01: How do input errors of mobility factors and zone data propagate in the calculation of production and attraction?

More formally: Independent errors: E_MOBI, E_ZDAT
Dependent errors: E_PROD, E_ATTR

The dependent input errors E_MOBI and E_ZDAT are fed into the regression analysis and used to explain the independent output errors E_PROD and E_ATTR.

Here are some cases showing the input error for E_MOBI and E_ZDAT varying from 0% to 20% with a step of 2%.

obs	E_MOBI	E_ZDAT	E_PROD	E_ATTR
1	0.000	0.000	0.000	0.000
2	0.000	0.020	0.012	0.012
3	0.000	0.040	0.023	0.023
4	0.000	0.060	0.035	0.035
5	0.000	0.080	0.046	0.046
6	0.000	0.100	0.058	0.058
7	0.000	0.120	0.069	0.069
8	0.000	0.140	0.081	0.081
9	0.000	0.160	0.092	0.092
10	0.000	0.180	0.104	0.104
11	0.000	0.200	0.115	0.115
12	0.020	0.000	0.012	0.012
Etc.				
110	0.180	0.200	0.155	0.155
111	0.200	0.000	0.115	0.115
112	0.200	0.020	0.116	0.116
113	0.200	0.040	0.118	0.118
114	0.200	0.060	0.121	0.121
115	0.200	0.080	0.124	0.124
116	0.200	0.100	0.129	0.129
117	0.200	0.120	0.135	0.135
118	0.200	0.140	0.141	0.141
119	0.200	0.160	0.148	0.148
120	0.200	0.180	0.155	0.155
121	0.200	0.200	0.163	0.163

The errors in PROD and ATTR are identical. It is correct, as they are calculated the same way.

Global statistics:

Obs	E_MOBI	E_ZDAT	E_PROD	E_ATTR
mean error	0.101	0.101	0.090	0.090
sigma error	0.063	0.063	0.035	0.035
sigma/mean	0.626	0.626	0.389	0.389
min error	0.000	0.000	0.012	0.012
max error	0.200	0.200	0.163	0.163
# obs	120	120	120	120

Both mean and sigma of the output error are lower than the input. The generation can be qualified as conservative, there will be no “bad surprises”.

Correlation Matrix:

	E_MOBI	*E_ZDAT*	*E_PROD*	*E_ATTR*
E_MOBI	1			
E_ZDAT	-0.02128	1		
E_PROD	0.678964	0.678964	1	
E_ATTR	0.678964	0.678964	1	1

Regression results for E_PROD and E_ATTR:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.99489909			
R Square	0.989824199			
Adjusted R Square	0.981263388			
Standard Error	0.009868573			
Observations	120			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	2	1.117841468	0.558920734	5739.069608
Residual	118	0.011491871	9.73887E-05	
Total	120	1.129333339		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_MOBI	0.438709263	0.010834031	40.49363127	4.66346E-71
E_ZDAT	0.438709263	0.010834031	40.49363127	4.66346E-71

The adjusted R^2 is 98%, all independent variables are significant and not strongly correlated amongst themselves, the regression equations are:

E_PROD = (0.439 * E_MOBI) + (0.439 * E_ZDAT)

E_ATTR = (0.439 * E_MOBI) + (0.439 * E_ZDAT)

From the analytical solution, we know that the error produced by the generation process on production and attraction values will be smaller than the combined errors of mobility factors and zone data. The generation is “conservative”.

Conclusion for the generation: The production or attraction error per zone will be less than proportional to the zone data and mobility errors and inversely proportional to the square root of the number of zone data types involved.

4.3.2 Distribution

T02: How do input errors of production, attraction and link time propagate in the calculation of trip value?

More formally: Independent errors: E_PROD, E_ATTR, E_TIME
Dependent error: E_TRIP

Global statistics:

Obs	E_PROD	E_ATTR	E_TIME	E_TRIP
mean error	0.100	0.100	0.100	0.202
sigma error	0.071	0.071	0.063	0.072
sigma/mean	0.705	0.705	0.630	0.357
min error	0.000	0.000	0.000	0.013
Max error	0.200	0.200	0.200	0.352
# obs	274	274	274	274

The mean error of E_TRIP is larger than the single errors of the independent variables. This means that if one or more of them has large errors, the effect on E_TRIP may be important.

Correlation Matrix:

	E_PROD	*E_ATTR*	*E_TIME*	*E_TRIP*
E_PROD	1			
E_ATTR	-0.00735	1		
E_TIME	-0.00823	-0.00823	1	
E_TRIP	0.664886	0.664886	0.191068	1

Regression results for E_TRIP:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.99369003			
R Square	0.987419876			
Adjusted R Square	0.983636997			
Standard Error	0.024208233			
Observations	274			
ANOVA				
	Df	*SS*	*MS*	*F*
Regression	3	12.46557793	4.155192642	7090.305828
Residual	271	0.158816451	0.000586039	
Total	274	12.62439438		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_PROD	0.795824759	0.017679194	45.01476474	8.5841E-128
E_ATTR	0.795824868	0.017679194	45.01477091	8.5838E-128
E_TIME	0.369234521	0.018846084	19.5921083	7.58553E-54

The adjusted R^2 is 98%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_TRIP = (0.796 * E_PROD) + (0.796 * E_ATTR) + (0.369 * E_TIME)

Conclusion for the distribution: The trip value error will be less than proportional to the individual production, attraction and link time error, but will be greater than their mean error.

4.3.3 Modal split

T03: How do input errors of link time, trip values and value of the exponent propagate in the calculation of trips?

More formally: Independent errors: E_TIME, E_TRIP
Independent variable: EXPO, α varying with values -6, -8 and -10
Dependent error: E_TR_A, ET_TR_B, for modes A and B

Global statistics:

Obs	E_TIME	E_TRIP	EXPO	E_TR_A	E_TR_B
mean error	0.101	0.101	-8.000	0.431	0.431
sigma error	0.063	0.063	1.635	0.249	0.249
sigma/mean	0.624	0.624	-0.204	0.577	0.577
min error	0.000	0.000	-10.000	0.020	0.020
max error	0.200	0.200	-6.000	1.017	1.017
# obs	360	360	360	360	360

Correlation Matrix:

	E_TIME	*E_TRIP*	*EXPO*	*E_TR_A*	*E_TR_B*
E_TIME	1				
E_TRIP	-2.6044E-17	1			
EXPO	0	0	1		
E_TR_A	0.91499145	9.3096E-18	-0.35764067	1	
E_TR_B	0.91499145	9.3096E-18	-0.35764067	1	1

Variable E_TRIP is not significant and E_TR_A and E_TR_B are linearly dependent. The regression will be done with E_TIME, EXPO as independent and E_TR_A as dependent.

Regression results for E_TR_A:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.984943274			
R Square	0.970113254			
Adjusted R Square	0.967236475			
Standard Error	0.086327409			
Observations	360			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	2	86.6012663	43.30063315	5810.276956
Residual	358	2.667966912	0.007452422	
Total	360	89.26923321		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TIME	3.330164775	0.068937181	48.30723747	6.7452E-159
EXPO	- 0.013405711	0.001003149	-13.36363527	2.51604E-33

The adjusted R^2 is 97%, all independent variables are significant (even EXPO with its small contribution) and not strongly correlated amongst themselves, the regression equation is:

E_TR_A = (3.33 * E_TIME) – (0.0134 * EXPO)

Conclusion for modal split: The trip value error will be 3 to 4 times the link time error and, to a small part, diminish with increasing exponent.

4.3.4 Assignment without calibration

All processes including an assignment procedure will be analyzed from two points of view: link flow and trip travel time. The link flow is the sum of all trips using that link on their path from origin to destination. The trip travel time is the sum of all travel times of the links of the path used by a trip from origin to destination.

T04: Analysis by links

How do input errors of link time and trip values propagate in the calculation of link flow values?

More formally: Independent errors: E_TIME, E_TRIP
Dependent error: E_QINI (initial link flows)

Global statistics:

obs	E_TIME	E_TRIP	E_QINI
mean error	0.100	0.110	0.064
sigma error	0.064	0.058	0.034
sigma/mean	0.635	0.525	0.525
min error	0.000	0.020	0.012
max error	0.200	0.200	0.117
# obs	110	110	110

Correlation Matrix:

	E_TIME	E_TRIP	E_QINI
E_TIME	1		
E_TRIP	-1.3022E-17	1	
E_QINI	1.8507E-17	1	1

The only correlation for E_QINI is with E_TRIP and a value = 1.0. E_QINI is proportional to E_TRIP.

Regression results for E_QINI, only to determine the proportionality factor to E_TRIP:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	1			
R Square	1			
Adjusted R Square	0.990825688			
Standard Error	2.90072E-07			
Observations	110			
ANOVA				
	Df	*SS*	*MS*	*F*
Regression	1	0.582409974	0.582409974	6.92179E+12
Residual	109	9.17143E-12	8.41415E-14	
Total	110	0.582409974		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TRIP	0.586351039	2.22868E-07	2630929.175	0

The adjusted R^2 is 99%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_QINI = (0.586 * E_TRIP)

This constant factor of 0.586 is the result of the formula for error addition applied to each link. For link a, the flow value QINI is the sum of all trips having link a on their path from origin to destination. As the formula for addition $dq = SQRT\{dx^2 + \dots dz^2 + du^2 + \dots + dw^2\}$ uses absolute error terms, we cannot simply calculate the constant factor analytically neither give a simple regression equation to estimate E_QINI.

However, what we can do is to give a rough estimate assuming that:

N = mean number of trips using a link (trips per link)

M = mean flow value of trips

Using the formula above, we have:

$dq = SQRT(N * (E\_TRIP * M)^2)$

q = N * M

E_QINI = dq / q = (SQRT(N) * E_TRIP * M) / (N * M)

or E_QINI = E_TRIP / SQRT(N)

From assignment, we know that for our model N = 6.8. Assuming a mean error E_TRIP of 10% we have E_QINI = 0.1 / SQRT(6.8) = 0.26 which is about half of the value from regression.

Conclusion for assignment (link flow): The link flow error is proportional to the trip value error and inversely proportional to the square root of the number of trips per link.

T05: Analysis by trips

How do input errors of link time and trip values influence the error of trip travel time?

More formally: Independent errors: E_TIME, E_TRIP
Dependent error: E_TRAV travel time

Global statistics:

Obs	E_TIME	E_TRIP	E_TRAV
Mean error	0.110	0.100	0.032
sigma error	0.058	0.064	0.017
sigma/mean	0.525	0.635	0.525
min error	0.020	0.000	0.006
max error	0.200	0.200	0.057
# obs	110	110	110

Correlation Matrix:

	E_TIME	*E_TRIP*	*E_TRAV*
E_TIME	1		
E_TRIP	1.3022E-17	1	
E_TRAV	1	0	1

The only correlation for E_TRAV is with E_TIME and a value = 1.0. E_TRAV is proportional to E_TIME.

Regression results for E_TRAV, only to determine the proportionality factor to E_TIME:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	1			
R Square	1			
Adjusted R Square	0.990825688			
Standard Error	3.02458E-07			
Observations	110			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	1	0.139882595	0.139882595	1.52909E+12
Residual	109	9.97143E-12	9.1481E-14	
Total	110	0.139882595		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TIME	0.287359221	2.32385E-07	1236563.428	0

The adjusted R^2 is 99%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_TRAV = (0.287 * E_TIME)

The situation here is similar to the analysis by links. This constant factor of 0.287 is the result of the formula for error addition applied to each trip. For trip t, the travel time value TRAV is the sum of all TIME values of links on the path from origin to destination. As the formula for addition $dq = SQRT\{dx^2 + \dots dz^2 + du^2 + \dots + dw^2\}$ uses absolute error terms, we cannot simply calculate the constant factor analytically nor give a simple regression equation to estimate E_TRAV.

However, what we can do is to give a rough estimate assuming that:

- N is the mean number of links used by a trip (links per trip),
- All links have the same mean time value M.

Using the formula above, we have:

E_TRAV = dt / t = (SQRT(N) * E_TIME * M) / (N * M)

or E_TRAV = E_TIME / SQRT(N)

From assignment, we know that for our model N = 16.7. Assuming a mean error E_TIME of 10% we E_TRAV = 0.1 / SQRT(16.7) = 0.41 which is about 140% the value from regression.

Conclusion for assignment (travel time): The travel time error is proportional to the link time error and inversely proportional to the square root of the number of links per trip.

Two general conclusions can be stated for error propagation during assignment:

The larger the matrix,

- the larger the number of trips per link, and the smaller the link flow error,

- the larger the number of links per trip, and the smaller the travel time error.

4.3.5 Assignment with multiplicative calibration (Entropy method)

T06: Analysis by links

How do input errors of time, trip values, measured flow and the percentage of measured links influence the error of calibrated link flow value?

More formally: Independent errors: E_TIME, E_TRIP, E_QMES
Independent variable: PMES
Dependent error: E_QCAL (calibrated link flow)

Global statistics:

Obs	E_TIME	E_TRIP	E_QMES	PMES	E_QCAL	R_QCAL
mean error	0.100	0.108	0.103	0.103	0.585	564%
sigma error	0.071	0.068	0.069	0.069	0.441	636%
sigma/mean	0.708	0.627	0.671	0.671	0.754	
min error	0.000	0.000	0.000	0.000	0.029	
max error	0.200	0.200	0.200	0.200	1.812	
# obs	580	580	580	580	580	

R_QCAL is the ratio of E_QCAL divided by the mean of the 4 independent values. As R_QCAL is around 600% it can have bad consequences for real cases. As input errors of 10% to 20% are quite possible, they will produce output errors of 60% to 120% for the calibrated flows on links.

Correlation Matrix:

	E_TIME	*E_TRIP*	*E_QMES*	*PMES*	*E_QCAL*
E_TIME	1				
E_TRIP	2.63E-17	1			
E_QMES	8.17E-17	-0.07937	1		
PMES	1.58E-17	-0.07937	0.015432	1	
E_QCAL	0.001669	0.189494	0.418171	0.006278	1

As E_TIME is not significant, the regression equation will be estimated without.

Regression results for E_QCAL:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.846230966			
R Square	0.716106847			
Adjusted R Square	0.713389713			
Standard Error	0.390948099			
Observations	580			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	3	222.4523169	74.15077228	485.1515986
Residual	577	88.18892018	0.152840416	
Total	580	310.641237		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TRIP	1.874249271	0.189591982	9.885699031	2.14862E-21
E_QMES	3.105552699	0.197073264	15.75836639	8.56999E-47
PMES	0.451825754	0.197073264	2.292679101	0.022225427

The adjusted R^2 is 71%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_QCAL = (1.87 * E_TRIP) + (3.11 * E_QMES) + (0.45 * PMES)

From the point of view of resource allocation, the best thing to do is to invest in lowering E_QMES, the error of section counts, as it has the strongest effect on E_QCAL.

Conclusion for multiplicative calibration (link flow): The mean and sigma of the calibrated link flow error are both about 6 times the individual input errors.

T07: Analysis by trips

How do input errors of time, trip values, measured flow and the percentage of measured links influence the error of the calibrated trip value?

More formally: Independent errors: E_TIME, E_TRIP, E_QMES
Independent variable: PMES
Dependent error: E_TRIC (calibrated trip value)

Global statistics:

Obs	E_TIME	E_TRIP	E_QMES	PMES	E_TRIC	R_TRIC
mean error	0.100	0.108	0.103	0.103	0.631	608%
sigma error	0.071	0.068	0.069	0.069	0.395	569%
sigma/mean	0.708	0.627	0.671	0.671	0.626	
min error	0.000	0.000	0.000	0.000	0.050	
max error	0.200	0.200	0.200	0.200	1.671	
# obs	580	580	580	580	580	

R_TRIC is the ratio of E_TRIC divided by the mean of the 4 independent values. It is about 600%, the same order as for R_QCAL, the calibrated link flow error. With input errors of 10% to 20%, which is plausible for real cases, the output errors for the calibrated values of the trip matrix will be in the range 60% to 120%.

Correlation Matrix:

	E_TIME	*E_TRIP*	*E_QMES*	*PMES*	*E_TRIC*
E_TIME	1				
E_TRIP	2.63E-17	1			
E_QMES	8.17E-17	-0.07937	1		
PMES	1.58E-17	-0.07937	0.015432	1	
E_TRIC	0.004	0.204008	0.54947	0.182389	1

As E_TIME is not significant, the regression equation will be estimated without.

Regression results for E_TRIC:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.911720487			
R Square	0.831234246			
Adjusted R Square	0.828912143			
Standard Error	0.306112146			
Observations	579			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	3	265.8414584	88.61381948	945.6715691
Residual	576	53.97387601	0.093704646	
Total	579	319.8153344		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TRIP	1.597949573	0.148473248	10.76254204	9.71403E-25
E_QMES	3.268756679	0.154336619	21.17939794	4.63447E-74
PMES	1.15043517	0.154336619	7.454064852	3.33843E-13

The adjusted R^2 is 83%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_TRIC = (1.36 * E_TRIP) + (2.98 * E_QMES) + (1.18 * PMES)

From the point of view of resource allocation, the best thing to do is to invest in lowering E_QMES, the measurement error of link flow.

Conclusion for multiplicative calibration (trip value): The mean and sigma of calibrated trip value errors are both about 6 times the individual input errors.

4.3.6 Assignment with additive calibration

To assure the comparability of the multiplicative and the additive method, both were applied with exactly the same input data. This explains why the global statistics of the input variables are identical.

T08: Analysis by links

How do input errors of time, trip values, measured flow and the percentage of measured links influence the error of calibrated link flow value?

More formally: Independent errors: E_TIME, E_TRIP, E_QMES
Independent variable: PMES
Dependent error: E_QCAL (calibrated link flow)

Global statistics:

Obs	E_TIME	E_TRIP	E_QMES	PMES	E_QCAL	R_QCAL
mean error	0.100	0.108	0.103	0.103	0.078	76%
sigma error	0.071	0.068	0.069	0.069	0.041	60%
sigma/mean	0.708	0.627	0.671	0.671	0.528	
min error	0.000	0.000	0.000	0.000	0.003	
max error	0.200	0.200	0.200	0.200	0.153	
# obs	580	580	580	580	580	

R_QCAL is about 1/10th of R_QCAL produced with the multiplicative method. The additive method seems to be very conservative.

Correlation Matrix:

	E_TIME	*E_TRIP*	*E_QMES*	*PMES*	*E_QCAL*
E_TIME	1				
E_TRIP	2.63E-17	1			
E_QMES	8.17E-17	-0.07937	1		
PMES	1.58E-17	-0.07937	0.015432	1	
E_QCAL	0.002206	0.940669	0.153698	0.039877	1

As E_TIME is not significant, the regression equation will be estimated without.

Regression results for E_QCAL:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.994239053			
R Square	0.988511295			
Adjusted R Square	0.986738371			
Standard Error	0.009511155			
Observations	580			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	3	4.491101972	1.497033991	16548.74709
Residual	577	0.052196617	9.04621E-05	
Total	580	4.543298589		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TRIP	0.571586912	0.004612476	123.921923	0
E_QMES	0.118698273	0.004794484	24.75725691	9.47716E-93
PMES	0.0499338	0.004794484	10.41484331	2.15946E-23

The adjusted R^2 is 99%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_QCAL = (0.563 * E_TRIP) + (0.0337 * E_QMES) + (0.0128 * PMES)

Conclusion for additive calibration (link flow): The mean and sigma of the calibrated link flow error are about 0.7 times the individual input errors.

T09: Analysis by trips

How do input errors of time, trip values, measured flow and the percentage of measured links influence the error of the calibrated trip value?

More formally: Independent errors: E_TIME, E_TRIP, E_QMES
Independent variable: PMES
Dependent error: E_TRIC (calibrated trip value)

Global statistics:

Obs	E_TIME	E_TRIP	E_QMES	PMES	E_TRIC	R_TRIC
mean error	0.100	0.108	0.103	0.103	0.133	129%
sigma error	0.071	0.068	0.069	0.069	0.071	102%
sigma/mean	0.708	0.627	0.671	0.671	0.529	
min error	0.000	0.000	0.000	0.000	0.005	
max error	0.200	0.200	0.200	0.200	0.260	
# obs	580	580	580	580	580	

R_TRIC is in the range 100% to 130%, the same order of the input errors. Here too, the additive method seems to be very conservative.

Correlation Matrix:

	E_TIME	*E_TRIP*	*E_QMES*	*PMES*	*E_TRIC*
E_TIME	1				
E_TRIP	2.63E-17	1			
E_QMES	8.17E-17	-0.07937	1		
PMES	1.58E-17	-0.07937	0.015432	1	
E_TRIC	0.002994	0.939238	0.159517	0.034886	1

As E_TIME is not significant, the regression equation will be estimated without.

Regression results for E_TRIC:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.994065165			
R Square	0.988165553			
Adjusted R Square	0.98639143			
Standard Error	0.016439609			
Observations	580			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	3	13.02088564	4.340295213	16059.65807
Residual	577	0.155940452	0.000270261	
Total	580	13.17682609		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TRIP	0.973297769	0.00797246	122.0824904	0
E_QMES	0.207685411	0.008287053	25.0614329	2.44679E-94
PMES	0.079156447	0.008287053	9.551821576	3.59446E-20

The adjusted R^2 is 99%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_TRIC = (0.962 * E_TRIP) + (0.0553 * E_QMES) + (0.0196 * PMES)

Conclusion for additive calibration (trip value): The mean and sigma of calibrated trip value errors are both about equal to the individual input errors.

4.3.7 Generation + Distribution + Modal Split + Assignment (4 Steps)

T10: Analysis by links

How do input errors of mobility factors, zone data, link time and modal split exponent propagate through the steps of the 4-step model?

More formally: Independent errors: E_MOBI, E_ZDAT, E_TIME
Independent variable: EXPO, α varying with values -6, -8 and -10
Dependent error: E_QINI (initial link flow)

Global statistics:

Obs	E_MOBI	E_ZDAT	E_TIME	EXPO	E_QINI
mean error	0.101	0.101	0.101	-8.000	0.275
sigma error	0.071	0.071	0.071	1.635	0.156
sigma/mean	0.700	0.700	0.700	-0.204	0.568
min error	0.000	0.000	0.000	-10.000	0.028
max error	0.200	0.200	0.200	-6.000	0.617
# obs	372	372	372	372	372

Correlation Matrix:

	E_MOBI	*E_ZDAT*	*E_TIME*	*EXPO*	*E_QINI*
E_MOBI	1				
E_ZDAT	-0.01639	1			
E_TIME	-0.01639	-0.01639	1		
EXPO	0	0	0	1	
E_QINI	0.059163	0.059163	0.918566	-0.29145	1

Regression results for E_QINI:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.987022222			
R Square	0.974212866			
Adjusted R Square	0.971285254			
Standard Error	0.051025077			
Observations	372			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	4	36.19644738	9.049111845	3475.670638
Residual	368	0.958109529	0.002603559	
Total	372	37.15455691		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_MOBI	0.041832049	0.036242337	1.154231555	0.249154262
E_ZDAT	0.041832049	0.036242337	1.154231555	0.249154262
E_TIME	1.913870662	0.036242337	52.80759519	8.0171E-174
EXPO	- 0.009937207	0.00078467	-12.66419205	9.30175E-31

The adjusted R^2 is 97%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_QINI = (0.042 * E_MOBI) + (0.042 * E_ZDAT) + (1.9 * E_TIME) – (0.01 EXPO)

Conclusion for the 4 step model (link value): The link value error will mainly depend on link time errors, less on mobility and zone data errors, and to a small part on the exponent of the modal split function.

T11: Analysis by trips

How do input errors of mobility factors, zone data, link time and modal split exponent propagate through the steps of the 4-step model?

More formally: Independent errors: E_MOBI, E_ZDAT, E_TIME
Independent variable: EXPO, α varying with values -6, -8 and -10
Dependent error: E_TRAV (trip travel time)

Global statistics:

Obs	E_MOBI	E_ZDAT	E_TIME	EXPO	E_TRAV
mean error	0.100	0.100	0.125	-8.000	0.036
sigma error	0.071	0.071	0.056	1.636	0.016
sigma/mean	0.708	0.708	0.448	-0.204	0.448
min error	0.000	0.000	0.050	-10.000	0.014
max error	0.200	0.200	0.200	-6.000	0.057
# obs	300	300	300	300	300

Correlation Matrix:

	E_MOBI	*E_ZDAT*	*E_TIME*	*EXPO*	*E_TRAV*
E_MOBI	1				
E_ZDAT	-7.4E-17	1			
E_TIME	2.49E-17	2.49E-17	1		
EXPO	0	0	0	1	
E_TRAV	-2.5E-17	-4.3E-17	1	0	1

E_TRAV is fully explained by E_TIME, whereas E_MOBI, E_ZDAT and EXPO are not correlated. The regression will be run with E_TIME only.

Regression results for E_TRIC:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	1			
R Square	1			
Adjusted R Square	0.996655518			
Standard Error	4.66469E-17			
Observations	300			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	1	0.464488704	0.464488704	2.13467E+32
Residual	299	6.50603E-31	2.17593E-33	
Total	300	0.464488704		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TIME	0.28736	1.9668E-17	1.46105E+16	0

The adjusted R^2 is 100% and the equation is

E_TRAV = (0.287 * E_TIME)

Conclusion for the 4 step model (trip travel time): The trip travel time of the 4 step model is proportional to the link travel time error.

4.3.8 Equilibrium Assignment

T12: Analysis by links

How do input errors of link travel time and trip value as well as the parameters of the BPR volume delay function propagate through equilibrium assignment?

More formally: Independent errors: E_TIME, E_TRIP
Independent variable: ALFA, BETA of the BPR function for link time T
$T = T0*(1+\alpha*(q/c)^{\beta})$ with q = flow and c = capacity
ALFA, varying with values 0.65, 0.90, 1.15, 1,40
BETA, varying with values 2, 3, 4, 5
Dependent error: E_QINI link flow, equilibrium assignment

Global statistics:

Obs	E_TIME	E_TRIP	ALFA	BETA	E_QINI
mean error	0.125	0.125	1.025	3.500	0.028
sigma error	0.056	0.056	0.280	1.120	0.013
sigma/mean	0.448	0.448	0.273	0.320	0.448
min error	0.050	0.050	0.650	2.000	0.011
max error	0.200	0.200	1.400	5.000	0.047
# obs	256	256	256	256	256

Correlation Matrix:

	E_TIME	*E_TRIP*	*ALFA*	*BETA*	*E_QINI*
E_TIME	1				
E_TRIP	-3E-16	1			
ALFA	-2.8E-17	5.72E-17	1		
BETA	0	0	0	1	
E_QINI	-0.00107	0.998902	-0.00043	-0.00227	1

As E_TIME, ALFA and BETA are not significant, only E_TRIP will be taken in the equation

Regression results for E_QINI:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.999816771			
R Square	0.999633575			
Adjusted R Square	0.995712007			
Standard Error	0.000585954			
Observations	256			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	1	0.238849169	0.238849169	695658.5179
Residual	255	8.75523E-05	3.43343E-07	
Total	256	0.238936721		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TRIP	0.223070042	0.00026745	834.0614593	0

The adjusted R^2 is 100% and the equation is

E_QINI = (0.223 * E_TRIP)

Conclusion for equilibrium assignment (link value): The link value error depends on the trip error.

T13: Analysis by trips

How do input errors of link travel time and trip value as well as the parameters of the BPR function propagate through equilibrium assignment?

More formally: Independent errors: E_TIME, E_TRIP
Independent variable: both parameters of the BPR function
ALFA, varying with values 0.65, 0.90, 1.15, 1,40
BETA, varying with values 2, 3, 4, 5
Dependent error: E_TRAV (trip travel time)

Global statistics:

Obs	E_TIME	E_TRIP	ALFA	BETA	E_TRAV
mean error	0.125	0.125	1.025	3.500	0.037
sigma error	0.056	0.056	0.280	1.120	0.016
sigma/mean	0.448	0.448	0.273	0.320	0.434
min error	0.050	0.050	0.650	2.000	0.014
max error	0.200	0.200	1.400	5.000	0.061
# obs	256	256	256	256	256

Correlation Matrix:

	E_TIME	*E_TRIP*	*ALFA*	*BETA*	*E_TRAV*
E_TIME	1				
E_TRIP	-3E-16	1			
ALFA	-2.8E-17	5.72E-17	1		
BETA	0	0	0	1	
E_TRAV	0.998719	0.025263	0.022418	0.020112	1

As E_TRIP, ALFA and BETA are not significant, only E_TIME will be taken in the equation:

Regression results for E_TRAV:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.99971889			
R Square	0.999437859			
Adjusted R Square	0.99551629			
Standard Error	0.000955507			
Observations	256			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	1	0.413921758	0.413921758	453367.6028
Residual	255	0.000232813	9.12994E-07	
Total	256	0.414154571		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TIME	0.29365575	0.000436127	673.3257776	0

The adjusted R^2 is 100% and the equation is

E_TRAV = (0.294 * E_TIME)

Conclusion for the equilibrium assignment (trip travel time): The trip travel time error of the equilibrium assignment is proportional to the link travel time error.

4.3.9 Comparison of two states S0 (actual) and S1 (future)

T14: Analysis by links

How do input errors of link travel time and trip value influence the error of the difference of link flow between two states S0 and S1?

More formally: Independent errors: E_TIME, E_TRIP, E_QINI and E_QCAL
Dependent error: E_QDEL link flow, difference

Global statistics:

Obs	E_TIME	E_TRIP	E_QINI	E_QCAL	E_QDEL
mean error	0.100	0.110	0.065	0.071	0.104
sigma error	0.064	0.058	0.034	0.037	0.055
sigma/mean	0.635	0.525	0.525	0.525	0.525
min error	0.000	0.020	0.012	0.013	0.019
max error	0.200	0.200	0.119	0.129	0.190
# obs	110	110	110	110	110

Correlation Matrix:

	E_TIME	*E_TRIP*	*E_QINI*	*E_QCAL*	*E_QDEL*
E_TIME	1				
E_TRIP	-1.3E-17	1			
E_QINI	-3.7E-18	1	1		
E_QCAL	0	1	1	1	
E_QDEL	1.37E-17	1	1	1	1

As E_TIME is not significant and all other variables are strongly correlated with E_TRIP, only E_TRIP will be taken in the equation

Regression results for E_QDEL:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	1			
R Square	1			
Adjusted R Square	0.990825688			
Standard Error	2.91873E-07			
Observations	110			
ANOVA				
	Df	*SS*	*MS*	*F*
Regression	1	1.527150924	1.527150924	1.79264E+13
Residual	109	9.28571E-12	8.519E-14	
Total	110	1.527150924		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TRIP	0.949476623	2.24253E-07	4233958.241	0

The adjusted R^2 is 99% and the equation is

$$E\_QDEL = (0.949 * E\_TRIP)$$

Conclusion for the comparison of two states S0 and S1 (link value): The error in difference of link values depends only the trip error.

T15: Analysis by trips

How do input errors of link travel time and trip value influence the error of the difference in travel time between two states S0 and S1?

More formally: Independent errors: E_TIME, E_TRIP, E_TRAV and E_TRAC
Dependent error: E_TDEL (difference of trip travel time)

Global statistics:

Obs	E_TIME	E_TRIP	E_TRAV	E_TRAC	E_TDEL
mean error	0.110	0.100	0.033	0.036	0.049
sigma error	0.058	0.064	0.018	0.019	0.026
sigma/mean	0.525	0.635	0.525	0.525	0.525
min error	0.020	0.000	0.006	0.007	0.009
max error	0.200	0.200	0.061	0.066	0.089
# obs	110	110	110	110	110

Correlation Matrix:

	E_TIME	*E_TRIP*	*E_TRAV*	*E_TRAC*	*E_TDEL*
E_TIME	1				
E_TRIP	1.3E-17	1			
E_TRAV	1	3.21E-17	1		
E_TRAC	1	3.29E-18	1	1	
E_TDEL	1	0	1	1	1

As E_TRIP is not significant and all other variables are strongly correlated with E_TIME, only E_TIME will be taken in the equation:

Regression results for E_TDEL:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	1			
R Square	1			
Adjusted R Square	0.990825688			
Standard Error	2.77604E-07			
Observations	110			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	1	0.337090094	0.337090094	4.37415E+12
Residual	109	8.4E-12	7.70642E-14	
Total	110	0.337090094		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TIME	0.446083636	2.1329E-07	2091445.737	0

The adjusted R^2 is 99% and the equation is

E_TDEL = (0.446 * E_TIME)

Conclusion for the comparison of two states S0 and S1 (trip travel time): The error in difference of trip travel times depends only the link travel time error.

4.4 Summary of the test results

Test#	indep. error	dep. error	min/step/max error range (%)	R^2(%)
T01, generation				
	E_MOBI, mobility factors		0 / 2 / 20%	
	E_ZDAT, zone data		0 / 2 / 20%	
		E_PROD, production		
		E_ATTR, attraction		98%
E_PROD = (0.439 * E_MOBI) + (0.439 * E_ZDAT)				
E_ATTR = (0.439 * E_MOBI) + (0.439 * E_ZDAT)				
T02, distribution				
	E_PROD, zone production		0 / 5 / 20%	
	E_ATTR, zone attraction		0 / 5 / 20%	
	E_TIME, link time		0 / 2 / 20%	
		E_TRIP, trip value		98%
E_TRIP = (0.796 * E_PROD) + (0.796 * E_ATTR) + (0.369 * E_TIME)				
T03, modal split				
	E_TIME, link time		0 / 2 / 20%	
	E_TRIP, trip value		0 / 2 / 20%	
	EXPO, exponent		-6, -8, -10	
		E_TR_A, trip value mode A		97%
		E_TR_B, trip value mode B		
E_TR_A = (3.33 * E_TIME) – (0.0134 * EXPO)				
T04/T05, assignment				
T04 analysis **by link**				
	E_TIME, link time		0 / 2 / 20%	
	E_TRIP, trip value		0 / 2 / 20%	
		E_QINI, link flow, initial		99%
E_QINI = (0.586 * E_TRIP)				
T05 analysis **by O / D** relation				
	E_TIME, link time		0 / 2 / 20%	
	E_TRIP, trip value		0 / 2 / 20%	
		E_TRAV, trip travel time		99%
E_TRAV = (0.287 * E_TIME)				

Test#	**indep. error**	**dep. error**	**min/step/max error range (%)**	**R^2(%)**
T06/T07, assignment with multiplicative calibration				
T06 analysis **by link**				
	E_TIME, link time		0 / 5 / 20%	
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link counts		0 / 5 / 20%	
	PMES, % counted links		0 / 5 / 20%	
		E_QCAL, link flow, calibr.		71%

E_QCAL = (1.87 * E_TRIP) + (3.11 * E_QMES) + (0.452 * PMES)

T07 analysis **by O / D** relation				
	E_TIME, link time		0 / 5 / 20%	
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link counts		0 / 5 / 20%	
	PMES, % counted links		0 / 5 / 20%	
		E_TRIC, trip value calibr.		83%

E_TRIC = (1.60 * E_TRIP) + (3.27 * E_QMES) + (1.15 * PMES)

T08/T09, assignment with additive calibration				
T08 analysis **by link**				
	E_TIME, link time		0 / 5 / 20%	
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link counts		0 / 5 / 20%	
	PMES, % counted links		0 / 5 / 20%	
		E_QCAL, link flow, calibr.		99%

E_QCAL = (0.571 * E_TRIP) + (0.119 * E_QMES) + (0.050 * PMES)

T09 analysis **by O / D** relation				
	E_TIME, link time		0 / 5 / 20%	
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link counts		0 / 5 / 20%	
	PMES, % counted links		0 / 5 / 20%	
		E_TRIC, trip value calibr.		99%

E_TRIC = (0.973 * E_TRIP) + (0.207 * E_QMES) + (0.079 * PMES)

T10/T11, generation + distribution + modal split + assignment				
T10 analysis by link				
	E_MOBI, mobility factors		0 / 5 / 20%	
	E_ZDAT, zone data		0 / 5 / 20%	
	E_TIME, link time		0 / 5 / 20%	
	EXPO, exponent		-6, -8, -10	
		E_QINI, link flow, initial		97%

E_QINI = (0.042 * E_MOBI) + (0.042 * E_ZDAT) + (1.91 * E_TIME) – (0.01 EXPO)

T11 analysis by O / D relation				
	E_MOBI, mobility factors		0 / 5 / 20%	
	E_ZDAT, zone data		0 / 5 / 20%	
	E_TIME, link time		0 / 5 / 20%	
	EXPO, exponent		-6, -8, -10	
		E_TRAV, trip travel time		100%

E_TRAV = (0.287 * E_TIME)

Test#	indep. error	dep. error	min/step/max error range (%)	R^2(%)
T12/T13, equilibrium assignment				
T12 analysis by link				
	E_TIME, link time		5 / 5 / 20%	
	E_TRIP, trip value		5 / 5 / 20%	
	ALFA, BPR function		0.65 / 0.25 / 1.40	
	BETA, BPR function		2.0 / 1.0 / 5.0	
		E_QINI, link flow, equilibrium		100%

E_QINI = (0.223 * E_TRIP)

Test#	indep. error	dep. error	min/step/max error range (%)	R^2(%)
T13 analysis by O / D relation				
	E_TIME, link time		5 / 5 / 20%	
	E_TRIP, trip value		5 / 5 / 20%	
	ALFA, BPR function		0.65 / 0.25 / 1.40	
	BETA, BPR function		2.0 / 1.0 / 5.0	
		E_TRAV, trip travel time, equilibrium		100%

E_TRAV = (0.294 * E_TIME)

Test#	indep. error	dep. error	min/step/max error range (%)	R^2(%)
T14/T15, comparison of two states S0 (actual) and S1 (future)				
T14 analysis by link				
	E_TIME, link time		2 / 2 / 20%	
	E_TRIP, trip value		2 / 2 / 20%	
		E_QDEL, link flow, difference		99%

E_QDEL = (0.949 * E_TRIP)

Test#	indep. error	dep. error	min/step/max error range (%)	R^2(%)
T15 analysis by O / D relation				
	E_TIME, link time		2 / 2 / 20%	
	E_TRIP, trip value		2 / 2 / 20%	
		E_TDEL, trip travel time, difference		99%

E_TDEL = (0.446 * E_TIME)

4.5 Comparison of results for both calibration methods

The global behavior of the output errors of calibration is analyzed here with the results of tests T06/T07 for the multiplicative and tests T08/T09 for the additive method. For each test we calculate the ratio by which the input error amplifies the output errors.

assignment with calibration:	**T06/T07, multipl.**	**T08/T09 additive**
analysis **by link**		
R_QCAL = E_QCAL / mean of all input errors	564%	76%
R_QCAL = E_QCAL / sigma of all input errors	636%	60%
analysis **by O / D** relation		
R_TRIC = E_TRIC / mean of all input errors	608%	129%
R_TRIC = E_TRIC / sigma of all input errors	569%	102%

An important conclusion is that the additive method produces much less variability in the output errors than the multiplicative method. This result could be very interesting if the quality of the calibration produced with the additive method can be compared favorably with the results produced by the multiplicative method. This question is addressed here.

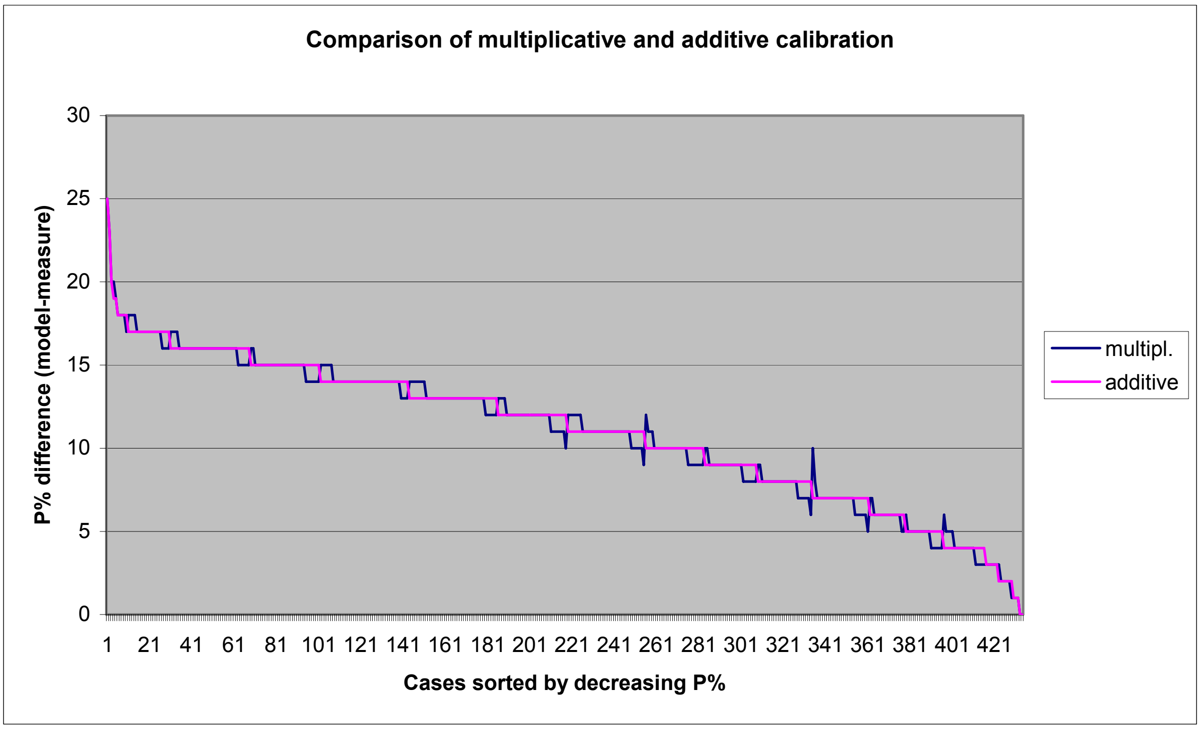
The results of a calibration are best described with a cumulative curve showing what 100-P% of links have less than P% difference between calculated and measured value. The lower the value P%, the higher the fit between calculus and measure will be. If 100% of the links have 0% difference, the fit is perfect.

NUMBER OF SECTIONS AS A FUNCTION OF THE DIFFERENCE
BETWEEN MEASUREMENT (MEAS) AND CALCULUS (CALC)
==

DIFFERENCE MEAS/CALC FROM %	TO %	NUMBER OF SECTIONS ABSOLUTE BEFORE CALIBRATION (0)	ABSOLUTE AFTER CALIBRATION (1)	RELATIVE BEFORE CALIBRATION (0)	RELATIVE AFTER CALIBRATION (1)	CUMULATED BEFORE CALIBRATION (0)	CUMULATED AFTER CALIBRATION (1)	0 20 40 60 80 100
0	0	25	93	8.6	31.8	8.6	31.8	. 0 . 1 . . . .
1	5	55	97	18.8	33.2	27.4	65.1	. . .0. . . .1. . . .
6	10	47	49	16.1	16.8	43.5	81.8	. . . . .0. . . 1 . .
11	15	49	28	16.8	9.6	60.3	91.4	. . . . . . 0 . . 1 .
16	20	30	8	10.3	2.7	70.5	94.2	. . . . 0 . .1.
21	25	22	6	7.5	2.1	78.1	96.2	. . . . 0 .1.
26	30	21	6	7.2	2.1	85.3	98.3	. . . . .0. 1
31	35	14	1	4.8	0.3	90.1	98.6	. . . . 0 1
36	40	4	1	1.4	0.3	91.4	99.0	. . . . 0 1
41	45	3	1	1.0	0.3	92.5	99.3	. . . . 0 1
46	50	2	-	0.7	0.0	93.2	99.3	01
51	55	7	-	2.4	0.0	95.5	99.3	01
56	60	1	-	0.3	0.0	95.9	99.3	01
61	65	1	1	0.3	0.3	96.2	99.7	01
66	70	3	-	1.0	0.0	97.3	99.7	01
71	75	3	-	1.0	0.0	98.3	99.7	. . . . *
76	80	2	-	0.7	0.0	99.0	99.7	. . . . *
81	85	-	-	0.0	0.0	99.0	99.7	. . . . *
86	90	1	1	0.3	0.3	99.3	100.0	. . . . *
91	95	-	-	0.0	0.0	99.3	100.0	. . . . *
96	100	1	-	0.3	0.0	99.7	100.0	. . . . *
101	...	1	-	0.3	0.0	100.0	100.0	. . . . *
TOTAL		292	292	100.0	100.0			

before calibration: 76% of the links have less than 24% difference
after calibration: 87% of the links have less than 13% difference

A new calibration method (here the additive one) can be compared with an established one (here the Entropy) by showing that the values P% are at least as good (= low) for the new one as for the established one. These data were collected during the 625 test runs of both calibration methods in parallel. As some cases had zero values for both methods, the total number of observations is somewhat lower than 625.



Comparison of multiplicative and additive calibration for the test case

The picture shows that there is no significant difference between the multiplicative (dark gray) and the additive (light gray) method.

This means that the additive calibration method, used under the same conditions and with the same data, produces results which are as good as the Entropy method. This is valid for means and standard deviations.

This is an important finding, it means that

The additive calibration method produces mean results that are as good as with the Entropy method but with output errors which are about 10 times smaller for link values and about 6 times smaller for trip values.

It is also interesting to notice that PMES has the smallest coefficient in the equation for the multiplicative method and is even about 10 times smaller for the additive one.

4.6 Summary of conclusions for the tests with the synthetic model

T01: Generation: The production or attraction error per zone are less than proportional to the zone data and mobility error.

T02: Distribution: The trip value error is more than proportional to the production, attraction and time error

T03: Modal split: The trip value error is 3 to 4 times the time error and, to a small part, decreases with increasing exponent.

T04: Assignment (link flow): The link flow error is about half of the trip value error and inversely proportional to the square root of the number of trips per link.

T05: Assignment (travel time): The travel time error is about 1/3 of the link time error and inversely proportional to the square root of the number of links per trip.

T06: Multiplicative calibration, Entropy (link flow): The link flow error is about 6 times the input error.

T07: Multiplicative calibration, Entropy (trip value): The calibrated trip value error is about 6 times the input error.

T08: Additive calibration (link flow): The calibrated link flow error is less than proportional to the input error. It is about 6 times less than with the multiplicative calibration.

T09: Additive calibration (trip value): The calibrated trip value error is of the same order as the input error. It is about 6 times less than with the multiplicative calibration

T10: Four step model (link flow): The link value error is about twice the input error and depends mainly on link time error, less on mobility and zone data error, and to a small part on the exponent of the modal split function.

T11: Four step model (travel time): The trip travel time error is about 1/3 of the link travel time error.

T12: Equilibrium assignment (link flow): The link value error is about ¼ of the trip error.

T13: Equilibrium assignment (travel time): The trip travel time error is about 1/3 of the link travel time error.

T14: Comparison of two states (link flow): The link value error is about the same as the trip error.

T15: Comparison of two states (travel time): The trip travel time error is about half of the link travel time error.

5 Tests with the Swiss Traffic Model (ARE)

5.1 Model setup

The model data were kindly given by the UVEK / ARE (Ministry of Transport, Dept for Area Development). The model consists of roughly 25000 links and 1200 zones. ERR_PROP was used to run the assignment and both calibration methods, multiplicative and additive. The results were analyzed with the same methodology as for the test case of the synthetic model.

As the number of count sections is given, the independent variable PMES is excluded from calculations. The results of the test model for PMES have shown that its coefficient is the smallest among all for the multiplicative method and still about 10 times smaller for the additive method. We can therefore assume that the exclusion of PMES will have no effect on the quality of the results.

5.2 Test runs

Each test is run with a new set of independent input error variables. Each dependent output error is analyzed as a function of all independent ones. The choice and combination of independent and dependent variables are summarized below for each test case:

Test#	**indep. error**	**dep. error**	**min/step/max error range (%)**	**R^2(%)**
T24/T25, assignment				
T24 analysis by link				
	E_TIME, link time		0 / 5 / 20%	
	E_TRIP, trip value		0 / 5 / 20%	
		E_QINI, link flow, initial		
T25 analysis by O / D relation				
	E_TIME, link time		0 / 5 / 20%	
	E_TRIP, trip value		0 / 5 / 20%	
		E_TRAV, trip travel time		
T26/T27, assignment and calibration, multiplicative (Entropy) method				
T26 analysis by link				
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link flow, counts		0 / 5 / 20%	
		E_QCAL, link flow, calibr.		
T27 analysis by O / D relation				
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link flow, counts		0 / 5 / 20%	
		E_TRIC, trip value calibr.		
T28/T29, assignment and calibration, additive method				
T28 analysis by link				
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link flow, counts		0 / 5 / 20%	
		E_QCAL, link flow, calibr.		
T29 analysis by O / D relation				
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link flow, counts		0 / 5 / 20%	
		E_TRIC, trip value calibr.		

5.3 Results

All tests will be analyzed from the two points of views of link flow and trip travel time. The flow of a link is the sum of all trips using it on their path from origin to destination. The travel time of a trip is the sum of all link travel times on the path from its origin to its destination.

5.3.1 Assignment without calibration

T24: Analysis by links

How do input errors of link time and trip values propagate in the calculation of link flow values?

More formally: Independent errors: E_TIME, E_TRIP
Dependent error: E_QINI (initial link flows)

Global statistics:

Obs	E_TIME	E_TRIP	E_QINI
Mean error	0.100	0.125	0.044
sigma error	0.073	0.057	0.020
sigma/mean	0.725	0.459	0.459
min error	0.000	0.050	0.017
Max error	0.200	0.200	0.071
# obs	20	20	20

Correlation Matrix:

	E_TIME	*E_TRIP*	*E_QINI*
E_TIME	1		
E_TRIP	1.1E-17	1	
E_QINI	-0.00451	0.999944	1

The only correlation for E_QINI is with E_TRIP. E_QINI is proportional to E_TRIP.

Regression results for E_QINI, only to determine the proportionality factor to E_TRIP:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.999990653			
R Square	0.999981306			
Adjusted R Square	0.947349727			
Standard Error	0.000213169			
Observations	20			
ANOVA				
	Df	*SS*	*MS*	*F*
Regression	1	0.046184055	0.046184055	1016350.316
Residual	19	8.63381E-07	4.54411E-08	
Total	20	0.046184918		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TRIP	0.350938	0.000348104	1008.142012	2.17783E-46

The adjusted R^2 is 95%, the regression equation is:

E_QINI = (0.351 * E_TRIP)

The conclusion is the same as for T4: The link flow error is proportional to the trip value error and inversely proportional to the square root of the number of trips per link. The proportionality factor is smaller than with the synthetic model because of the larger number of trips per link.

T25: Analysis by trips

How do input errors of link time and trip values influence the error of trip travel time?

More formally: Independent errors: E_TIME, E_TRIP
Dependent error: E_TRAV (travel time)

Global statistics:

Obs	E_TIME	E_TRIP	E_TRAV
mean error	0.125	0.100	0.039
sigma error	0.057	0.073	0.018
sigma/mean	0.459	0.725	0.459
Min error	0.050	0.000	0.016
Max error	0.200	0.200	0.062
# obs	20	20	20

Correlation Matrix:

	E_TIME	*E_TRIP*	*E_TRAV*
E_TIME	1		
E_TRIP	0	1	
E_TRAV	1	-1.8E-17	1

The only correlation for E_TRAV is with E_TIME. E_TRAV is proportional to E_TIME.

Regression results for E_TRAV, only to determine the proportionality factor to E_TIME:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	1			
R Square	1			
Adjusted R Square	0.947368421			
Standard Error	2.80976E-07			
Observations	20			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	1	0.03610868	0.03610868	4.57377E+11
Residual	19	1.5E-12	7.89474E-14	
Total	20	0.03610868		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_TIME	0.310306	4.58831E-07	676296.2478	4.2895E-100

The adjusted R^2 is 95%, the regression equation is:

E_TRAV = (0.310 * E_TIME)

Here too, the conclusion is the same as for T5: The travel time error is proportional to the link time error and inversely proportional to the square root of the number of links per trip. The proportionality factor is smaller than with the synthetic model because of the larger number of links per trip.

5.3.2 Assignment with multiplicative calibration (Entropy method)

T26: Analysis by links

How do input errors of trip values and measured flow influence the error of calibrated link flow value?

More formally: Independent errors: E_QMES, E_TRIP
Dependent error: E_QCAL (calibrated link flow)

Global statistics:

Obs	E_QMES	E_TRIP	E_QCAL	R_QCAL
mean error	0.104	0.104	0.277	266%
sigma error	0.071	0.071	0.117	165%
sigma/mean	0.678	0.678	0.421	
min error	0.000	0.000	0.067	
max error	0.200	0.200	0.490	
# obs	24	24	24	

Conclusion for multiplicative calibration (link flow): The mean and sigma of the calibrated link flow error are about 2 ½ time and 1 ½ times the individual input errors.

Correlation Matrix:

	E_QMES	*E_TRIP*	*E_QCAL*
E_QMES	1		
E_TRIP	-0.09091	1	
E_QCAL	0.844327	0.388378	1

Regression results for E_QCAL:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.993190561			
R Square	0.98642749			
Adjusted R Square	0.940356012			
Standard Error	0.03651421			
Observations	24			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	2	2.131824686	1.065912343	799.4617364
Residual	22	0.029332325	0.001333288	
Total	24	2.161157011		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_QMES	1.64223432	0.079998625	20.5282818	7.72131E-16
E_TRIP	0.95128792	0.079998625	11.89130337	4.73377E-11

The adjusted R^2 is 94%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_QCAL = (1.64 * E_QMES) + (0.951 * E_TRIP)

From the point of view of resource allocation, the first priority is to invest in lowering E_QMES and the second in lowering E_TRIP.

T27: Analysis by trips

How do input errors of trip values and measured flow influence the error of the calibrated trip value?

More formally: Independent errors: E_TRIP, E_QMES
Dependent error: E_TRIC (calibrated trip value)

Global statistics:

Obs	E_QMES	E_TRIP	E_TRIC	R_TRIC
mean error	0.104	0.104	0.302	290%
sigma error	0.071	0.071	0.132	187%
sigma/mean	0.678	0.678	0.437	
min error	0.000	0.000	0.065	
max error	0.200	0.200	0.533	
# obs	24	24	24	

Conclusion for multiplicative calibration (trip value): The mean and sigma of the calibrated link flow error are about 3 times and twice the individual input errors.

Correlation Matrix:

	E_QMES	*E_TRIP*	*E_TRIC*
E_QMES	1		
E_TRIP	-0.09091	1	
E_TRIC	0.901637	0.292915	1

Regression results for E_TRIC:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.994827864			
R Square	0.989682478			
Adjusted R Square	0.943758955			
Standard Error	0.034877725			
Observations	24			
ANOVA				
	Df	*SS*	*MS*	*F*
Regression	2	2.567080401	1.283540201	1055.147508
Residual	22	0.026762025	0.001216456	
Total	24	2.593842427		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_QMES	1.9356332	0.076413267	25.33111432	9.07206E-18
E_TRIP	0.8922508	0.076413267	11.67664773	6.70549E-11

The adjusted R^2 is 94%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_TRIC = (1.94 * E_QMES) + (0.892 * E_TRIP)

The conclusions are similar as for T26: From the point of view of resource allocation, the first priority is to invest in lowering E_QMES and the second in lowering E_TRIP.

5.3.3 Assignment with additive calibration

To assure the comparability of the multiplicative and the additive method, both were applied with exactly the same input data. This is why the output mean and sigma of the input variables are the same

T28: Analysis by links

How do input errors of trip values and measured flow influence the error of calibrated link flow value?

More formally: Independent errors: E_TRIP, E_QMES
Dependent error: E_QCAL (calibrated link flow)

Global statistics:

Obs	E_QMES	E_TRIP	E_QCAL	R_QCAL
mean error	0.104	0.104	0.046	44%
sigma error	0.071	0.071	0.027	38%
sigma/mean	0.678	0.678	0.592	
min error	0.000	0.000	0.001	
max error	0.200	0.200	0.088	
# obs	24	24	24	

Conclusion for additive calibration (link flow): The mean and sigma of the calibrated link flow error are both around 40% of the individual input errors.

Correlation Matrix:

	E_QMES	*E_TRIP*	*E_QCAL*
E_QMES	1		
E_TRIP	-0.09091	1	
E_QCAL	0.155236	0.948043	1

Regression results for E_QCAL:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.994424373			
R Square	0.988879835			
Adjusted R Square	0.942919827			
Standard Error	0.005835857			
Observations	24			
ANOVA				
	df	*SS*	*MS*	*F*
Regression	2	0.066629154	0.033314577	978.1939159
Residual	22	0.000749259	3.40572E-05	
Total	24	0.067378414		
	Coefficients	*Standard Error*	*T Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_QMES	0.08256112	0.012785723	6.457289874	1.69849E-06
E_TRIP	0.36196152	0.012785723	28.30982014	8.43032E-19

The adjusted R^2 is 94%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_QCAL = (0.0826 * E_QMES) + (0.362 * E_TRIP)

The relative weight of the coefficients is reversed compared to the multiplicative method. It confirms that the influence of E_QMES is much lower for the additive method than for the multiplicative one.

T29: Analysis by trips

How do input errors of trip values and measured flow influence the error of the calibrated trip value?

More formally: Independent errors: E_TRIP, E_QMES
Dependent error: E_TRIC (calibrated trip value)

Global statistics:

Obs	E_QMES	E_TRIP	E_TRIC	R_TRIC
mean error	0.104	0.104	0.120	115%
sigma error	0.071	0.071	0.072	102%
sigma/mean	0.678	0.678	0.603	
min error	0.000	0.000	0.002	
max error	0.200	0.200	0.229	
# obs	24	24	24	

The mean and sigma of the calibrated trip value error are about the same as the individual input errors.

Correlation Matrix:

	E_QMES	*E_TRIP*	*E_TRIC*
E_QMES	1		
E_TRIP	-0.09091	1	
E_TRIC	0.083838	0.973754	1

Regression results for E_TRIC:

SUMMARY OUTPUT				
Regression Statistics				
Multiple R	0.997095882			
R Square	0.994200198			
Adjusted R Square	0.948482025			
Standard Error	0.01108041			
Observations	24			
ANOVA				
	Df	*SS*	*MS*	*F*
Regression	2	0.463014918	0.231507459	1885.616326
Residual	22	0.002701061	0.000122775	
Total	24	0.465715979		
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
E_QMES	0.16117648	0.024275962	6.639344731	1.1243E-06
E_TRIP	0.99720928	0.024275962	41.07805419	2.71471E-22

The adjusted R^2 is 95%, all independent variables are significant and not strongly correlated amongst themselves, the regression equation is:

E_TRIC = (0.161 * E_QMES) + (0.997 * E_TRIP)

As for T28, E_TRIP has a much greater weight than E_QMES. It again confirms that the influence of E_QMES is much lower for the additive method than for the multiplicative one.

5.3.4 Summary of test results

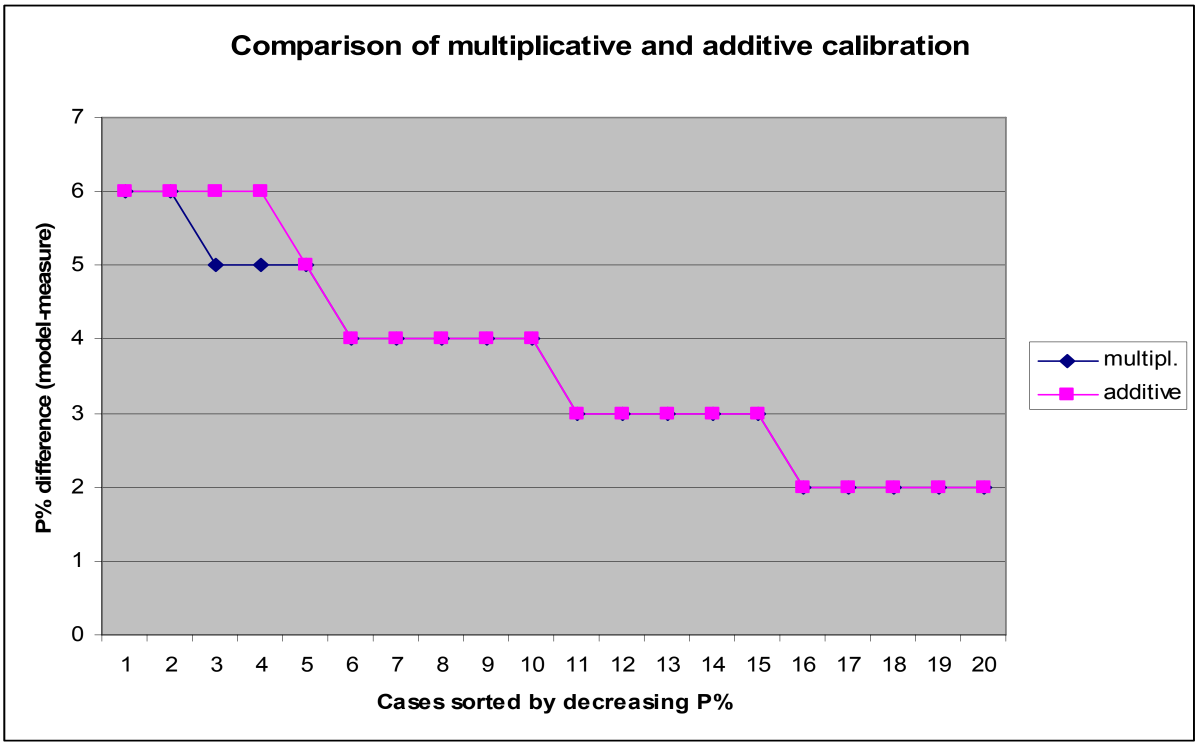
Test#	indep. error	dep. error	min/step/max error range (%)	R^2(%)
T24/T25, assignment				
T24 analysis by link				
	E_TIME, link time		0 / 5 / 20%	
	E_TRIP, trip value		0 / 5 / 20%	
		E_QINI, link flow, initial		95%
E_QINI = (0.351 * E_TRIP)				
T25 analysis by O / D relation				
	E_TIME, link time		0 / 5 / 20%	
	E_TRIP, trip value		0 / 5 / 20%	
		E_TRAV, trip travel time		95%
E_TRAV = (0.310 * E_TIME)				
T26/T27, assignment and calibration, multiplicative (Entropy) method				
T26 analysis by link				
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link flow, counts		0 / 5 / 20%	
		E_QCAL, link flow, calibr.		94%
E_QCAL = (1.64 * E_QMES) + (0.951 * E_TRIP)				
T27 analysis by O / D relation				
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link flow, counts		0 / 5 / 20%	
		E_TRIC, trip value calibr.		94%
E_TRIC = (1.94 * E_QMES) + (0.892 * E_TRIP)				
T28/T29, assignment and calibration, additive method				
T28 analysis by link				
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link flow, counts		0 / 5 / 20%	
		E_QCAL, link flow, calibr.		94%
E_QCAL = (0.0826 * E_QMES) + (0.362 * E_TRIP)				
T29 analysis by O / D relation				
	E_TRIP, trip value		0 / 5 / 20%	
	E_QMES, link flow, counts		0 / 5 / 20%	
		E_TRIC, trip value calibr.		95%
E_TRIC = (0.161 * E_QMES) + (0.997 * E_TRIP)				

5.3.5 Comparison of calibration methods for the Swiss model

A separate analysis was done for the global behavior of the output errors of calibration. For each case we calculate the ratio by which the output error is amplified from input errors.

assignment with calibration:	**T26/T27, multipl.**	**T28/T29 additive**
analysis **by link**		
ratio: mean E_QCAL / mean input errors	266%	44%
ratio: sigma E_QCAL / sigma input errors	165%	38%
analysis **by O / D** relation		
ratio: mean E_TRIC / mean input errors	290%	115%
ratio: sigma E_TRIC / sigma input errors	287%	102%

As in the synthetic case, the additive method produces much less variability in the output errors than the multiplicative method. And here too this result is only of some value if the quality of the calibration produced with the additive method can be compared favorably with the results produced by the multiplicative method.



Comparison of multiplicative and additive calibration for the Swiss model

The pictures above shows that it is the case: there is no significant difference between the results of both methods at the level of differences between calculus and measure. The finding of the synthetic model holds therefore equally well for the Swiss model:

The additive calibration method produces mean results as good as with the Entropy method but with output errors which are about 6 times smaller for link values and 3 times smaller for trip values.

5.4 Conclusion for the tests with the Swiss model

T24: Assignment (link flow): The link flow error is less than proportional to the trip value error and inversely proportional to the square root of the number of trips per link.

T25: Assignment (travel time): The travel time error is less than proportional to the link time error and inversely proportional to the square root of the number of links per trip.

T26: Multiplicative calibration, Entropy (link flow): The mean and sigma of the calibrated link flow error are about 2 ½ and 1 ½ times the input error. This is less than for the synthetic model but will still have bad consequences for real cases.

T27: Multiplicative calibration, Entropy (trip value): Here too, the mean and sigma of the calibrated trip value error are about 3 times the input error. It is of the same order as for the calibrated link flow error.

T28: Additive calibration (link flow): The mean and sigma of the calibrated link flow error is about 40% of the input error. This is about 1/5th of the error produced with the multiplicative method.

T29: Additive calibration (trip value): The mean and sigma of the calibrated trip value error are of the same order as the input error. It is about 1/3rd of the error produced with the multiplicative method.

Assignment in general: The larger the matrix,

- the larger the number of trips per link, and the smaller the link flow error will be
- the larger the number of links per trip, and the smaller the travel time error will be

Calibration in general: The additive calibration method produces mean results which are as good as with the Entropy method but with output error which are about 3 to 6 times smaller for link and trip values.

Resources allocation: Resources should be allocated in priority to decrease the input error of trip values and link travel time, as these two have the strongest influence on output error of link flow and trip time.

6 Error Propagation for Quality Management

The integration of explicit error propagation into transport modeling software opens new possibilities for online quality management, quality assessment, quality control, etc.

The idea is simple: if the quality (= errors) of the input is known and if the software is able to propagate this quality (= errors) throughout the whole calculation process, the output quality (= errors) will be known too.

Of course, the quality assessment of simulation software is not new. The traditional method uses the so called “brute force” approach with repeated simulations.

Input	Process	Output
X	Y = f(X)	Y
X_i distributed with σ_x	$Y_i = f(X_i)$, means	Y_i distributed with σ_y

This methodology has many disadvantages, it takes effort, time and costs to setup the experiment, to create software for the handling of input and output data, to run the needed number of experiments and eventually to analyze the results with multivariate statistical tools. The consequence is that this technique is rarely applied and will never get widely spread.

The possibility to separate and follow error propagation calculations in parallel to the normal processes simplifies many aspects:

Input	Process	Output
X	Y = f(X)	Y
σ_x	$\sigma_y = f(\sigma_x)$	σ_y

Where $\sigma_y = f(\sigma_x)$ are all the rules governing error propagation. The advantages are clear:

- minimal software extension thanks to operator overloading (like in ADA or C++),
- marginal increase of runtime, no need to run lots of cases,
- no need for statistical tools and analysis, the results are directly available.

The application for online traffic modeling is straightforward. The input errors are the σ_{qm} of the counted flows Qm and the σ_{vm} of the measured speeds Vm on links m equipped with sensors (loops, cameras, etc.). The output errors are the σ_{qa} of the calculated flow Qa and the σ_{va} of the calculated speeds Va on all links of the network.

Input, on “measured” links m Actual values of Q and V	Process dynamic traffic model	Output, on all links a estimated situation
Qm	Qa = f(Qm, Vm)	Qa
σ_{qm}	$\sigma_{qa} = f(\sigma_{qm}, \sigma_{vm})$	σ_{qa}
Vm	Va = f(Qm, Vm)	Va ➔ Ta ➔ T_travel
σ_{vm}	$\sigma_{va} = f(\sigma_{qm}, \sigma_{vm})$	σ_{va} ➔ σ_{ta} ➔ σ_{t_travel}

The advantages are many:

- continuous quality control of input, process and output,
- input: targeted elimination of weak points in the measuring infrastructure,
- process: choice of algorithms that minimize the effects of error propagation,
- output: guaranteed quality of the traffic situation, information and travel times.

7 Conclusions

The results for the four step model show that generation, distribution and assignment do not amplify input errors, whereas modal split can multiply the input error by a factor of 3 to 4.

The two input variables having the most influence on output error are trip values and link travel times. The errors of trip values influence the errors of link values and the errors of link travel times influence the errors of trip travel time. Resources should therefore be allocated in priority to decrease the input errors of trip values and link travel times.

The major finding of the study concerns the two calibration methods. One of them is the classical Entropy method of Willumsen [Ortuzar, Willumsen]. The other one is its additive form developed by [de Rham 87]. Both methods produce best possible estimates for link loads, but the second one with much less distortion of the resulting distance distribution.

For the test case, the comparison of both calibration methods led to the discovery that the additive method produces output errors that are about 10 times smaller for link flow and 6 times smaller for trip travel time than with the classical method.

This finding was confirmed with the Swiss ARE model, where the additive method produces output errors that are about 6 times smaller for link flow and 3 times smaller for trip travel time than with the classical method.

The difference is due to the error propagation formula for addition, where the error diminishes proportionally to the square root of the number of terms.

There are good reasons to recommend the use of the additive calibration method instead of the multiplicative one:

the results at the level of the link flows are equivalent

there is less distortion of the trip matrix

the output errors of link flow are not greater than input errors of trips

the output errors of trip travel times are about half of the input errors of link times

the allocation of resources to diminish trip value and link time errors has a higher benefit/cost ratio: the investment will not be destroyed by the bad behavior in error propagation of the classical Entropy method.

8 Future research

Here are some ideas and suggestions for further research. It concerns the application of error propagation methods to parent fields of transport modeling.

Multinomial logit models: This type of model could not be analyzed in depth in spite of its importance for practice.

Robustness of the method: This analysis was done with one small synthetic and one large real model. How robust are the results? What are the conclusions for other type of models?

Stated Preference: It should be possible to estimate the output errors of stated and/or revealed preference surveys and feed them as input errors into transport models. The analysis of the resulting output errors could be used as feedback to improve the next SP/RP surveys.

Public transport: The effect of error propagation in systems depending on time tables and other time constraints seems not to have been analyzed so far. Which assignment model, which calibration algorithm is optimal from the point of view of error propagation?

Floating Car Data: It is only a question of a few years for FCD to play a major role for incident detection, traffic monitoring, etc. Why not analyze how FCD errors will influence the results of online transport models?

Short and long term forecasting: What is the effect of input errors on the precision of short and long term forecasts?

Future research should be concentrated on topics where the use of error propagation will simplify procedures by avoiding the great number of runs necessary to insure statistical significance.

9 Glossaries

Fractional error terms used in the regression analyses:

E_ATTR	zone attraction
E_MOBI	mobility factor
E_PROD	zone production
E_QCAL	link flow, calibrated
E_QDEL	link flow, difference
E_QINI	link flow, initial
E_QMES	link flow, counted
E_TDEL	trip travel time, difference
E_TIME	link time
E_TR_A	resulting trip value mode A
E_TR_B	resulting trip value mode B
E_TRAC	trip travel time calibrated
E_TRAV	trip travel time
E_TRIC	trip value, calibrated
E_TRIP	trip value, initial
E_ZDAT	zone data
R_QCAL	ratio of E_QCAL by the mean of E_TIME, E_TRIP, E_QMES and PMES
R_TRIC	ratio of E_TRIC by the mean of E_TIME, E_TRIP, E_QMES and PMES

Institutions and associations

ARE	Bundesamt für Raumentwicklung
ASTRA	Bundesamt für Strassen
BAV	Bundesamt für Verkehr
EPFL	Ecole Polytechnique Fédérale de Lausanne
ETH	Eidgenössische Technische Hochschule Zürich
SVI	Vereinigung Schweizerischer Verkehrsingenieure
TBA	Tiefbauamt
UVEK	Eidg. Departement für Umwelt, Verkehr, Energie und Kommunikation
VSS	Schweizerischer Verband der Strassen- und Verkehrsfachleute

Other abbreviations:

ADA	Programming language
ALFA	BPR function, alpha = 0.15 (default value)
ARCS	number of arcs on a path from O to D
BETA	BPR function, beta = 4 (default value)
BPR	Bureau of Public Roads
D	zone, destination
EXPO	Exponent of modal split function
FCD	Floating Car Data
NTRI	number of trips passing through a link
O	zone, origin
PMES	percentage of counted links
SE	System Equilibrium
SQRT	square root
UE	User Equilibrium

Test cases

Test	Model	Method	Analysis
T01	test	Generation	production/attraction
T02	test	Distribution	by O/D
T03	test	modal split	by O/D
T04	test	Assignment	by link
T05	test	Assignment	by O/D
T06	test	assignment + calibration multiplicative	by link
T07	test	assignment + calibration multiplicative	by O/D
T08	test	assignment + calibration additive	by link
T09	test	assignment + calibration additive	by O/D
T10	test	gener. + distrib. + mod-spl. + assignment	by link
T11	test	gener. + distrib. + mod-spl. + assignment	by O/D
T12	test	equilibrium assignment	by link
T13	test	equilibrium assignment	by O/D
T14	test	difference of two assignments	by link
T15	test	difference of two assignments	by O/D
T24	ARE	Assignment	by link
T25	ARE	Assignment	by O/D
T26	ARE	assignment + calibration multiplicative	by link
T27	ARE	assignment + calibration multiplicative	by O/D
T28	ARE	assignment + calibration additive	by link
T29	ARE	assignment + calibration additive	by O/D

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